DECAY MODES OF SPIN-TWO MESONS*

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Recent evidence indicates the existence of a nonet of $J^P=2^+$ mesons. These are

$$K^*(1430):\quad T=\tfrac{1}{2},\ Y=\pm 1,$$
$$A_2(1320):\quad T=1,\ Y=0,$$
$$f(1250):\quad T=0,\ Y=0,$$
$$f'(1525):\quad T=0,\ Y=0.$$

In this note, we compare the observed partial decay widths of these states with theoretical predictions based on SU(3). The results strongly support the assignment of these states to the reducible $1\oplus 8$ representation of SU(3) with considerable f,f' mixing. Certain remarkable regularities characterize the nonets of $J^P=1^-$ and $J^P=2^+$ mesons, and we present a theoretical framework from which these regularities may be understood.[1]

A recent determination of the partial decay widths of the A_2, $K^*(1430)$, and $f(1250)$ may be found in the preceding paper.[2] These eight $J^P=2^+$ mesons generally are not assigned to an irreducible unitary octet, since their masses do not satisfy the Gell-Mann–Okubo formula, but they are attractive candidates for a reducible nonet.[1] We speculate that the remaining $T=Y=0$ member of the nonet is the recently discovered[3] f' at 1525 MeV. The physical f and f' are thus regarded as linear combinations of the unitary singlet f_1 and the $T=Y=0$ member of the unitary octet f_8. The mixing angle θ_2 is determined[4] in terms of the observed masses under the hypothesis that mass splitting transforms like hypercharge under SU(3). We obtain[5]

$$\sin^2\theta_2=(\hat{f}'-\hat{f}_8)(\hat{f}'-\hat{f})^{-1},$$

where $\hat{f}_8=[4\hat{K}^*(1430)-\hat{A}_2]/3$ is the square of the mass of the f_8 which would satisfy the Gell-Mann–Okubo formula. This yields $\theta_2\approx 30°$, so that

$$f'\approx\tfrac{1}{2}\sqrt{3}f_8-\tfrac{1}{2}f_1,$$
$$f\approx\tfrac{1}{2}f_8+\tfrac{1}{2}\sqrt{3}f_1.$$

Consider the decays of the $J^P=2^+$ mesons into two pseudoscalar mesons. We assume that the coupling constants are given by exact SU(3), so that there are only two relevant couplings which conserve C:

$$(6)^{1/2}F\,\mathrm{Tr}(T_8\{P_8,P_8\})+Gf_1\,\mathrm{Tr}(P_8P_8),\tag{1}$$

where P_8 is the usual traceless 3×3 matrix describing the pseudoscalar octet and T_8 is the corresponding traceless 3×3 matrix describing the $J^P=2^+$ octet. Since the amount of mixing is determined, we may express all of the coupling constants of A_2, $K^*(1430)$, and the physical f and f' to two $J^P=0^-$ mesons in terms of the two parameters F and G. Table I gives the predicted partial decay widths which result when the experimental values

$$\Gamma(f\to 2\pi)=100\text{ MeV and }\Gamma(A_2\to K+\bar{K})=6\text{ MeV}$$

are used as input. We have assumed simple p^5/M^2 phase space, as is appropriate for these $l=2$ decay modes when SU(3) is applied to their relativistic matrix elements and no structure is assumed. (M is the mass of the decaying state and p is the c.m. decay momentum.)

Some decays of the $J^P=2^+$ mesons into a vector meson and a pseudoscalar meson are kinematically allowed. There is just one SU(3)-invariant C-conserving coupling,

$$H\,\mathrm{Tr}(T_8[V_8,P_8]),$$

where V_8 is the traceless 3×3 matrix representing the vector-meson octet. The decay widths predicted in Table I are based on the input

$$\Gamma(A_2\to\rho+\pi)=70\text{ MeV}$$

M517 1-5

Table I. Decays of the $J^P=2^+$ nonet.

Decay mode[a]	Rate divided by phase space: General form[b]	Rate divided by phase space: Form in terms of F^2, H^2 only[c]	Phase space[d]	Predicted rate (MeV)[e]	Observed rate (MeV)[f]
$f\to\pi+\pi$	$3(2F\sin\theta+G\cos\theta)^2$	$35.7F^2$	53.6	<u>100</u>	100
$f\to K+\bar{K}$	$4(F\sin\theta-G\cos\theta)^2$	$15.2F^2$	5.3	2.4	<5
$f\to\eta+\eta$	$(2F\sin\theta-G\cos\theta)^2$	$2.1F^2$	1.5	0.2	N.I.
$A_2\to\pi+\eta$	$8F^2$	$8F^2$	25.2	11	5 ± 2
$A_2\to K+\bar{K}$	$12F^2$	$12F^2$	9.2	<u>6</u>	6 ± 2
$K^{**}\to K+\pi$	$18F^2$	$18F^2$	45.1	42	75
$K^{**}\to K+\eta$	$2F^2$	$2F^2$	13.7	1.4	<38
$f'\to\pi+\pi$	$3(2F\cos\theta-G\sin\theta)^2$	$0.30F^2$	101.8	1.7[g]	N.I.
$f'\to K+\bar{K}$	$4(F\cos\theta+G\sin\theta)^2$	$20.8F^2$	28.4	31	seen
$f'\to\eta+\eta$	$(2F\cos\theta+G\sin\theta)^2$	$9.9F^2$	17.9	9	N.I.
$A_2\to\rho+\pi$	$4H^2$	$4H^2$	13.2	<u>70</u>	68 ± 8
$K^{**}\to K^*+\pi$	$1.5H^2$	$1.5H^2$	13.3	26	25 ± 25
$K^{**}\to\rho+K$	$1.5H^2$	$1.5H^2$	3.9	8	<6
$K^{**}\to\omega+K$	$1.5\sin^2\varphi H^2$	$0.62H^2$	3.1	2.5	<25
$f'\to(K^*+\bar{K}+\bar{K}^*+K)$	$6\cos^2\theta H^2$	$4.5H^2$	2.8	17	seen

[a]All charge states are included. K^{**} is called $K^*(1430)$ in the text.
[b]θ and φ are called θ_2 and θ_1 in the text.
[c]We have set $\theta=30°$, $\varphi=40°$, and $G=2\sqrt{2}F$ (see text).
[d]Our phase space is p^5/M^2 (in units of 10^{-3} BeV^3) for the two-pseudoscalar-meson decays and p^5 (in units of 10^{-3} BeV^5) for the other decays.
[e]Input values are underlined. Two distinct solutions for F and G will fit the input data; the one with $F/G<0$ is totally unacceptable and has not been displayed.
[f]We assume total widths of f and K^{**} are 100 MeV, and A_2 is 80 MeV. The branching fractions were provided by Janos Kirz, private communication. The quoted errors do not include uncertainties in the total widths. N.I. means no information is available.
[g]The degree of suppression of $f'\to\pi+\pi$ is extremely sensitive to the input.

and p^5 phase space. The decay $K^*(1430)\to K+\omega$ has been calculated using an ω-φ mixing angle $\theta_1=40°$. Agreement with experiment, both for vector-meson plus pseudoscalar-meson decays and for two-pseudoscalar-meson decays, is quite satisfying.

In Fig. 1, we display the well-established meson states and most of the recently observed ones. (We have omitted certain low-lying and very speculative states which are sometimes assigned to $J^P=0^+$ multiplets.) There is a pseudoscalar nonet (with an η-η' mixing angle $\theta_0=10°$), a vector nonet, and a spin-two nonet. We also display 16 states for which the $J^P=1^+$ assignment is often proposed but in no case has been confirmed; we suspect that eventually a $J^P=1^+$ nonet will emerge.

We wish to call attention to certain remarkable similarities between the $J^P=1^-$ nonet and the new $J^P=2^+$ nonet:

(1) The mass spectra of the two nonets are similar. In both nonets the $T=\frac{1}{2}, Y=\pm1$ states are heavier than the $T=1, Y=0$ states; in both nonets there is considerable mixing between

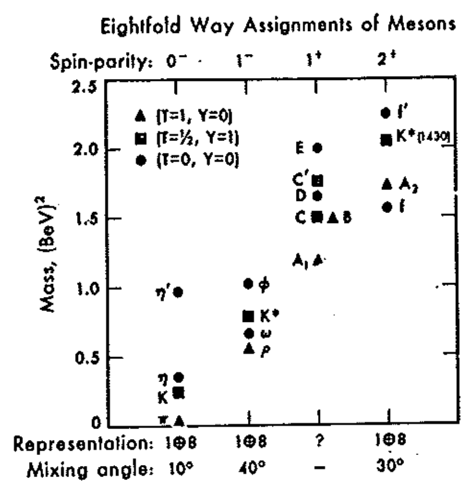

FIG. 1. In addition to the three meson nonets, with $J^P=0^-$, 1^-, and 2^+, which are discussed in the text, 16 possible candidates for $J^P=1^+$ assignments are shown. These are $A_1(1090)$, $B(1215)$, $C(1215)$, $D(1280)$, $C'(1330)$, and $E(1410)$. The B and A_1 have different G parities and must, if they exist, belong to different SU(3) multiplets. D and E have the same G parity (if their isospins are correct) and may mix.

M517 2-5

the two $T=Y=0$ states, and the mixing angles are approximately equal ($\theta_1=40°$ for $J^P=1^-$, and $\theta_2=30°$ for $J^P=2^+$); and the heaviest state in each nonet is the $T=Y=0$ state which is mostly octet.

(2) The masses of the particles in each nonet accurately satisfy a mass formula which was first put forward by Schwinger[6] for the vector-meson nonet,[5]

$$(\hat{\varphi}-\hat{\omega}_8)(\hat{\omega}-\hat{\omega}_8)=-(8/9)(\hat{K}^*-\hat{\rho})^2,$$

where $\hat{\omega}_8=(4\hat{K}^*-\hat{\rho})/3$. The corresponding formula for the $J^P=2^+$ mesons,

$$(\hat{f}'-\hat{f}_8)(\hat{f}-\hat{f}_8)=-(8/9)[\hat{K}^*(1430)-\hat{A}_2]^2,$$

would be identically satisfied if the mass of the f' were 1510 MeV.[7]

(3) The decay modes $\varphi\to\rho+\pi$ and $f'\to\pi+\pi$ are analogs. In both cases, the mostly octet $T=Y=0$ member of the nonet decays via D-type and singlet couplings into two $T=1, Y=0$ mesons. Both decay modes appear to be suppressed. It is known[8] that $\Gamma(\varphi\to\rho+\pi)\sim 0.07\,\Gamma(\omega\to 3\pi)$, even though phase space favors $\varphi\to\rho+\pi$. There is no experimental evidence for $f'\to 2\pi$, and our analysis here predicts $\Gamma(f'\to\pi+\pi)\lesssim 0.2\,\Gamma(f\to\pi+\pi)$, where phase space again favors $f'\to\pi+\pi$.

Okubo[9] first suggested a method of dealing with the vector-meson nonet. He introduces the 3×3 matrix

$$V_9=V_8+3^{-1/2}\omega_1\underline{1},$$

and requires that, when the Lagrangian is expressed in terms of V_9, the factor $\mathrm{Tr}(V_9)$ never appears. Thus, the most general $1\oplus 8$ mass Lagrangian for the vector mesons satisfying Okubo's criterion is

$$a\,\mathrm{Tr}(V_9V_9)+b\,\mathrm{Tr}(V_9\lambda_8V_9),$$

where λ_8 is the usual 3×3 matrix representing hypercharge in SU(3). The resulting mass formulas, $\hat{\rho}=\hat{\omega}$ and $\hat{\rho}+\hat{\varphi}=2\hat{K}^*$, are crudely satisfied, and the mixing angle relating ω and φ to ω_1 and ω_8 is given by $\sin\theta_C=1/\sqrt{3}$, or $\theta_C\approx 35.3°$. Moreover, there is a unique vector-meson–vector-meson–pseudoscalar-meson coupling given by $\mathrm{Tr}(\{V_9,V_9\}P_8)$, which rigorously forbids the decay mode $\varphi\to\rho+\pi$.

We can similarly place the nine $J^P=2^+$ mesons in a 3×3 matrix,

$$T_9=T_8+3^{-1/2}f_1\underline{1}.$$

The same mass Lagrangian as above, except with T_9 substituted for V_9, yields the mass formulas $\hat{f}=\hat{A}_2$, and $\hat{A}_2+\hat{f}'=2\hat{K}^*(1430)$, and predicts the same mixing angle θ_C. These results are again in crude agreement with experiment. The coupling of $J^P=2^+$ mesons to two pseudoscalar mesons, if we do not allow $\mathrm{Tr}T_9$ to appear as a factor, forbids $f'\to\pi+\pi$.

Okubo's mass Lagrangian neglects two terms compatible with the usual assumptions about the SU(3) symmetry breaking. The full mass Lagrangian for a meson nonet may be written

$$a\,\mathrm{Tr}(M_9M_9)+b\,\mathrm{Tr}(M_9\lambda_8M_9)+c(\mathrm{Tr}M_9)^2+d\,\mathrm{Tr}M_9\,\mathrm{Tr}(M_9\lambda_8), \qquad (2)$$

where M_9 stands for either V_9 or T_9. If we set $d=0$ but keep $c\neq0$, Okubo's two mass formulas reduce to one, the Schwinger formula. The physical masses of the particles in a nonet determine the four coefficients. We obtain (in BeV^2)

$$a=0.723,\quad b=-0.424,\quad c=0.022,\quad d=-0.014\quad (M_9=V_9),$$
$$a=1.944,\quad b=-0.606,\quad c=-0.067,\quad d=-0.055\quad (M_9=T_9).$$

Thus for both the $J^P=1^-$ and the $J^P=2^+$ mesons, we get the same hierarchy of terms.

Just as the corrections to Okubo's mass Lagrangian are found to be small, we may also suppose that the corrections to his interaction Lagrangian are small. Then an appealing way to calculate the couplings of nonets of mesons is to admit only couplings which do not contain $\mathrm{Tr}M_9$ as a factor, but to use the physical mixing angles.[10] The 3×3 matrix for the $J^P=2^+$ mesons is then

$$T_9=\begin{pmatrix} \dfrac{f\cos(\theta_C-\theta_2)+f'\sin(\theta_C-\theta_2)+A_2^0}{\sqrt{2}} & A_2^+ & K^{*+}(1430) \\ A_2^- & \dfrac{f\cos(\theta_C-\theta_2)+f'\sin(\theta_C-\theta_2)-A_2^0}{\sqrt{2}} & K^{*0}(1430) \\ K^{*-}(1430) & \bar{K}^{*0}(1430) & [f\sin(\theta_C-\theta_2)-f'\cos(\theta_C-\theta_2)] \end{pmatrix},$$

M517 3-5

and the coupling which governs the decays of the $J^P=2^+$ mesons into two pseudoscalar mesons is $\mathrm{Tr}T_9\{P_8,P_8\}$. The coupling constants in (1) should therefore satisfy $G=2\sqrt{2}F$. But this ratio is given in terms of our two inputs:

$$G/(2\sqrt{2}F)=(0.338)[\Gamma(f\to\pi+\pi)/\Gamma(A_2\to K+\bar{K})]^{1/2}-(6)^{-1/2}=0.97. \quad (3)$$

Thus the inputs, considering their large uncertainties, are entirely consistent with the prescription which altogether neglects the coupling $\mathrm{Tr}T_9\ \mathrm{Tr}(P_8P_8)$. Furthermore, since the right-hand side of (3) is so close to 1.00, the rates for decay into two pseudoscalar mesons, calculated on the basis of two inputs in Table I, are essentially unchanged if we use $\mathrm{Tr}T_9\{P_8,P_8\}$ and only one input.

As a further consequence of neglecting couplings which contain $\mathrm{Tr}T_9$ as a factor, we note that the decay modes $f'\to\pi+\pi$, $f'\to4\pi$, $f'\to\rho+\rho$, and $f'\to A_2+\pi$ should all be strongly suppressed.

The form of the coupling of two vector mesons and a pseudoscalar meson is, correspondingly, taken to be $\mathrm{Tr}V_9\{V_9,P_8\}$, where V_9 is obtained from T_9 by replacing $[A_2,K^*(1430),f,f']$ by $(\rho,K^*,\omega,\varphi)$ and θ_2 by θ_1. This gives the decay rate $\Gamma(\varphi\to\rho+\pi)$ in terms of $\Gamma(\omega\to3\pi)$, when the latter rate is calculated[11] from a $\rho\pi$ intermediate state and the known $\rho\pi\pi$ coupling strength. We obtain

$$\Gamma(\varphi\to\rho+\pi;\text{three charge modes})\approx17\tan^2(\theta_C-\theta_1)\Gamma(\omega\to3\pi).$$

For $\Gamma(\varphi\to\rho+\pi)=0.56\pm0.25$ MeV,[8] $\Gamma(\omega\to3\pi)=8.1\pm1.6$ MeV, we obtain (as one root) $\theta_1=39°\pm1°$, which is to be compared with the value $\theta_1=40°$ inferred from the masses.

Finally, we may conjecture that the couplings of the pseudoscalar nonet to other particles are analogously restricted, in spite of the fact that this nonet does not satisfy the Schwinger formula.[12] We construct P_9 from T_9 above by replacing $[A_2,K^*(1430),f,f']$ by $[\pi,K,\eta'(960),\eta]$ and θ_2 by $\theta_0=\pm10°$. We then allow only those couplings in which $\mathrm{Tr}P_9$ does not appear as a factor. Then the rate for $A_2\to\eta'(960)+\pi$ is given in terms of the others:

$$\begin{aligned}\Gamma[A_2\to\eta'(960)+\pi]&=24\cos^2(\theta_C-\theta_0)F^2(1.1)\\&=1.1\text{ MeV }(\theta_0=+10°)\\&=0.7\text{ MeV }(\theta_0=-10°).\end{aligned}$$

The rates for decays with an η in the final state are not altogether insensitive to this introduction of pseudoscalar singlet coupling; for example, the rate for $A_2\to\eta+\pi$ in Table I should be multiplied by $3\sin^2(\theta_C-\theta_0)$, which is 0.56 for $\theta_0=+10°$. The rate for $f'\to\eta'(960)+\eta$ is totally negligible (about 2 keV).

We may use this nonet coupling scheme to relate the electromagnetic decays involving one vector meson and one pseudoscalar meson.[13] The conventional SU(3) coupling may be written

$$\alpha\,\mathrm{Tr}(V_9\{P_9,Q\})+\beta\,\mathrm{Tr}V_9\,\mathrm{Tr}P_9Q+\gamma\,\mathrm{Tr}P_9\,\mathrm{Tr}V_9Q,$$

where Q is the usual traceless 3×3 matrix representing the electromagnetic field. Our scheme tells us to keep only the first term. 12 electromagnetic decays are then given in terms of a single coupling constant, when simple p^3 phase space is assumed. Table II lists the predictions obtained from the single input $\Gamma(\omega\to\pi^0+\gamma)=1.0$ MeV. The relatively large branching ratios for $\varphi\to\eta+\gamma$ and for $\eta'(960)\to\rho^0+\gamma$ should be capable of confirmation.

The nonet coupling scheme has no new predictions for the decays of $J^P=2^+$ mesons into a vector meson plus a pseudoscalar meson,

Table II. Electromagnetic decays of $J^P=0^-$ and 1^- mesons.

Decay mode	Predicted rate[a] (keV)
$\rho^0\to\pi^0+\gamma$	100
$\rho^+\to\pi^++\gamma$	100
$\rho^0\to\eta+\gamma$	20(60)
$\omega\to\pi^0+\gamma$	1000[b]
$\omega\to\eta+\gamma$	1(6)
$K^{*+}\to K^++\gamma$	60
$K^{*0}\to K^0+\gamma$	230
$\varphi\to\pi+\gamma$	10
$\varphi\to\eta+\gamma$	330(200)
$\varphi\to\eta'+\gamma$	0.2(0.8)
$\eta'\to\rho+\gamma$	250(140)
$\eta'\to\omega+\gamma$	24(16)

[a]Where two predictions appear, the set in parentheses is appropriate to $\theta_0=-10°$, the other set to $\theta_0=+10°$.

[b]Input.

M517 4-5

since

$$\mathrm{Tr}(T_9[V_9, P_9]) = \mathrm{Tr}(T_8[V_8, P_8]).$$

This is a general result, whenever the octet couplings are F type.

Note added in proof.—The authors apologize for quoting experimental results which are not always in exact agreement with the preceding two papers. Such are the perils of working with unpublished data. The theoretical predictions agree as well with the published values of masses and decay widths as with the values previously available to the authors.

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[1]An octet of $J^P = 2^+$ mesons has been suggested often. Cf. S. L. Glashow, Proceedings of the International School of Physics "Enrico Fermi," Varenna Lectures, 1964 (Academic Press, Inc., New York, to be published); E. Borchi and R. Gatto, Phys. Letters 14, 352 (1965); Hong-Mo Chan, to be published; G. A. Ringland and E. J. Squires, to be published; M. Suzuki, to be published; F. Von Hippel, private communication. R. Delbourgo, M. A. Rashid, and J. Strathdee, Phys. Rev. Letters 14, 719 (1965), and R. C. Hwa and S. H. Patil, to be published, consider octet-singlet mixing and estimate several branching ratios.

[2]S. U. Chung, O. I. Dahl, L. M. Hardy, R. I. Hess, L. D. Jacobs, J. Kirz, and D. H. Miller, preceding Letter [Phys. Rev. Letters 15, 325 (1965)]. This work contains references to earlier experimental papers on these particles.

[3]V. E. Barnes, B. B. Culwick, P. Guidoni, G. R. Kalbfleisch, G. W. London, R. B. Palmer, D. Radojičić, D. C. Rahm, R. R. Rau, C. R. Richardson, N. P. Samios, J. R. Smith, B. Goz, N. Horwitz, T. Kikuchi, J. Leitner, and R. Wolfe, preceding Letter [Phys. Rev. Letters 15, 322 (1965)].

[4]J. J. Sakurai, Phys. Rev. Letters 9, 472 (1962).

[5]A circumflex above a particle name indicates the square of its mass.

[6]J. Schwinger, Phys. Rev. 135, B816 (1964).

[7]The Schwinger formula for the vector mesons reappears in an SU(6) theory when the SU(3)-octet mass splitting is assumed to transform like a member of the adjoint representation of SU(6). The derivation [M. A. Baqi Beg and V. Singh, Phys. Rev. Letters 13, 418 (1964)] would not apply to the $J^P = 2^+$ nonet, however.

[8]James Lindsey, private communication.

[9]S. Okubo, Phys. Letters 5, 165 (1963).

[10]Note that a mass spectrum determines a mixing angle only within a sign, and allows a second value for the coefficient d in (2). We choose the signs of θ_1 and θ_2 (implied by the quoted values of d) which suppress $\varphi \to \rho + \pi$ and $f' \to \pi + \pi$.

[11]M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962). The φ decay was first discussed by J. J. Sakurai (reference 4) and was reconsidered recently by Joel Yellin, to be published.

[12]The coefficients of (2) for $M_9 = P_9$ are (in BeV^2) $a = 0.169$, $b = -0.450$, $c = 0.244$, $d = 0.153$ or 0.447.

[13]Cf. S. L. Glashow, Phys. Rev. Letters 11, 48 (1963), and Proceedings of the International School of Physics "Enrico Fermi," Varenna Lectures 1964 (Academic Press, Inc., New York, to be published); as well as V. V. Anisovich, A. A. Anselm, Ya. I. Azimov, O. S. Danilov, and I. T. Dyatlov, Phys. Letters 16, 196 (1965), for earlier discussions of the electromagnetic decays of vector mesons. Electromagnetic decays of the $\eta'(960)$ have been treated by S. K. Kundu and D. C. Peaslee, Nuovo Cimento 36, 277 (1965); S. Boudier and C. Bouchiat, Phys. Letters 15, 96 (1965); and R. H. Dalitz and D. G. Sutherland, to be published.