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Good enough tools for global warming policy making

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We present a simple analysis of the global warming problem caused by the emissions of CO_2 (a major greenhouse gas) into the atmosphere resulting from the burning of fossil fuels. We provide quantitative tools which enable policymakers and interested citizens to explore the following issues central to the global warming problem.

(i) At what rate are we permitted to continue to emit CO_2 after the global average atmospheric concentration has 'stabilized' at some chosen target level? The answer here provides *the magnitude of the effort*, measured by the necessary total reduction of today's global (annual) emissions rate to achieve stabilization. We shall see that stabilized emissions rates for all interesting stabilized concentration levels are *much lower* than the current emissions rate, but these small finite values are very important.

(ii) Across how many years can we spread the total effort to reduce the annual CO_2 emissions rate from its current high value to the above-mentioned low and stabilized target value? The answer here provides *the time-scale of the total mitigation effort* for any chosen atmospheric concentration target level. We confirm the common understanding that targets below a doubling of the pre-industrial concentration create great pressure to produce action immediately, while targets above double the pre-industrial level can tolerate longer periods of inaction.

(iii) How much harder is the future mitigation effort, if we do not do our share of the job now? Is it a good idea to overshoot a stabilization target? The quantitative answers here provide *the penalty of procrastination*. For example, the mitigation task to avoid doubling the pre-industrial level is a problem that can be addressed gradually, over a period extending more than a century, if started immediately, but procrastination can turn the effort into a much more urgent task that extends over only a few decades. We also find that overshooting target levels is a bad idea.

The quality of public discourse on this subject could be much enhanced if ball-park quantitative answers to these questions were more widely known.

Keywords: mitigation; headroom; wedges; overshoot; stabilization; procrastination

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1. Introduction: the need for simple quantitative tools

The climate problem is an unprecedented challenge to humanity. It is global in scope, its time-scale is centuries, and the mitigation strategies required are often fraught with risks as large as the problem itself. The spirit of this paper is similar to the paper by Stephen Pacala and one of us (R.H.S.), which introduced two simple concepts, the 'stabilization triangle' and the 'stabilization wedge' (Pacala & Socolow 2004; see also, Socolow & Pacala 2006). These concepts have enabled a more participatory and inclusive discussion of climate change mitigation. Here, we provide simple tools to connect fossil-fuel CO_2 emissions to their consequences for the atmospheric CO_2 concentration. Our goal is to promote public discussion of the magnitude of the total effort, the mitigation time frame, the penalties of procrastination, and the desired minimum long-term emissions rates after stabilization. The recent heroic, sustained effort in carbon-cycle research has produced many insights into the mechanisms responsible for the movements of CO_2 after it enters the atmosphere (Wigley 1991, 1993; Intergovernmental Panel on Climate Change 2001; Friedlingstein *et al.* 2006). The tools we provide are abstractions of these insights. As with the 'wedges' papers, our goal is to provide tools that are 'good enough' to generate meaningful numbers that are useful to policy makers and the interested public.

All discussions of climate mitigation policy today work within the frame of reference provided by the 1992 United Nations Framework Convention on Climate Change (United Nations 1992). The Framework Convention asserts that the world's management goal is to 'prevent dangerous anthropogenic interference with the climate system'. In discussions of CO_2, this management goal has been understood to require that, over the long-term, the quantity of carbon as CO_2 in the atmosphere should reach a steady-state (stabilization) value, and that, thereafter, the amount of carbon in the atmosphere should not wander far from this target. The lower the stabilization target, the tighter the constraints on future emissions.

We investigate stabilization targets, representative of the current policy discussion. Current discussion focuses on limiting the risk of crossing one or more (unknown) 'tipping points' that could trigger some undesirable climate change instabilities. Examples include the shutting down of the North Atlantic thermohaline circulation and the melting of the Antarctic and Greenland ice sheets. Our colleague, Steve Pacala, calls these speculative outcomes 'monsters behind the door', each rattling its own doorknob. Experts argue about how fearsome these monsters actually are and at what range of atmospheric concentrations they are likely to emerge. The world's political leaders discuss, implicitly, how large a risk of living amidst these monsters the world should tolerate.

The world's leaders have not yet chosen a target. However, many of them have articulated the goal of avoiding doubling the 'pre-industrial' quantity of CO_2 in the atmosphere and are asking how far below this benchmark, the target should be set (Stern 2007). (Here, pre-industrial refers to a long period before the industrial revolution, i.e. roughly the thousand years before 1800.) Others are implicitly advocating much higher targets by endorsing the delay of aggressive mitigation for many decades, as we will explain in §6. Today's concentration is already roughly 'one-third of the way towards doubling'. In the present paper, we

shall use 'doubling' as one of the benchmark stabilization targets in numerical calculations. Everyone should understand that the amount of atmospheric CO_2 the world can tolerate is still under debate (Wigley 2004).

The tools we provide in this paper are applicable to any stabilization target. A simple formula is developed that connects the lowest stabilization target the world can achieve at any given time to the strongest effort the world can sustain from that time forward. Numerical results are presented for several targets and mitigation strategies.

The carbon content of the global atmosphere at time t (in years) is denoted by $C(t)$, which is in units of billions of metric tons of carbon (GtC), and the global annual rate of CO_2 emission to the atmosphere is denoted by $E(t)$ in units of GtC yr^{-1}. The current year is denoted by t_0.

The usefulness of the tools we are introducing in this paper does not depend on exactly which categories of CO_2 emissions, or exactly which greenhouse gases, are included in $E(t)$. Our $E(t)$ follows the Oak Ridge database (Marland *et al.* 2006), which does not include deliberate deforestation, responsible today for about 20% as much transfer of CO_2 to the atmosphere as the extraction and burning of fossil fuels. It does not include other land-use issues that result in a transfer of CO_2 between the biosphere and the atmosphere, such as those associated with deliberate afforestation. It *does* include cement emissions—today, about 4% of fossil fuel emissions. This database has not yet had to confront CO_2 capture and storage (CCS), which decouples fossil fuel use from CO_2 emissions to the atmosphere and which may become an important carbon mitigation strategy (Socolow 2005). Our $E(t)$ also does not explicitly include other greenhouse gases, such as methane and nitrous oxide. Our tools easily accommodate other definitions of $E(t)$.

For further details about the Earth's environment and useful relationships between its properties, see appendix A. For quantitative exercises elucidating the carbon cycle that share much of the motivation of this paper, see Harte (1988).

(*a*) *The Rosetta stone*

In the literature, the amount of carbon in the atmosphere is usually represented by its concentration expressed in units of parts per million (p.p.m.). However, the common choice for the amount of carbon coming into or going out of the atmosphere is expressed in units of billions of tons of carbon per year (GtC yr^{-1}). In the present paper, we represent the atmospheric concentration by the atmospheric carbon content. Conversion between these two units is straightforward and is expressed in words as follows: *adding 2.1 billion metric tons of carbon to the atmosphere as carbon dioxide results in the increase of the fraction of the atmosphere's molecules that are CO_2 molecules by one part in a million.* Since, this equality connects the language of one community with the language of another, we call '2.1 GtC = 1 p.p.m.' the Rosetta stone.[1] Occasionally, the equivalent atmospheric CO_2 concentration in p.p.m. is reported in parentheses after the GtC value. We always use GtC yr^{-1} as the unit of $E(t)$, the annual emissions rate. We discuss the Rosetta stone further in appendix A.

[1] Discourse about carbon mitigation is further complicated by the wide use of two units: 'tons of carbon' and 'tons of carbon dioxide'. Since the atomic weight of carbon is 12 and the atomic weight of oxygen is 16, each CO_2 molecule is 12/44 carbon by weight. Accordingly, an equivalent statement of the Rosetta stone is: 7.7 $GtCO_2 = 1$ p.p.m.

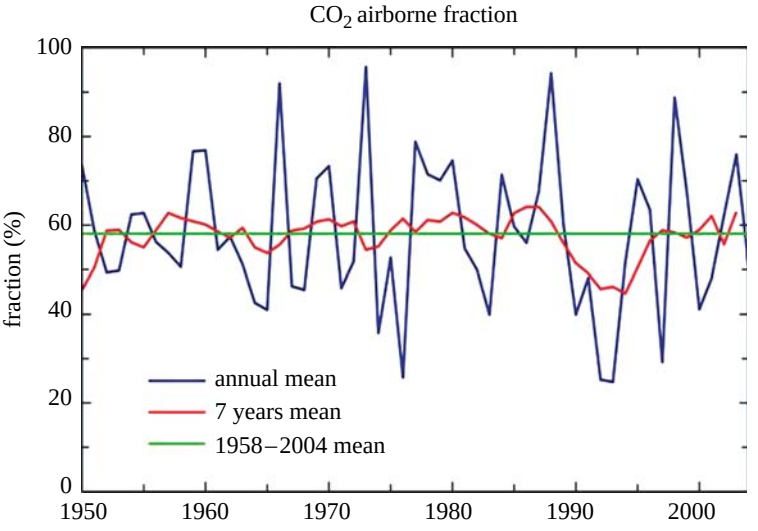

Figure 1. Historical observational data of the fraction: annual increase of the carbon content of the atmosphere divided by the quantity of carbon in annual emissions of CO_2 from fossil fuels (Hansen & Sato 2004; updated in Hansen 2006).

2. The *constant airborne fraction model* and the total emissions reduction effort

The goal of this paper is to provide simple tools for global warming policy makers. In this section, we put forth several empirical relations between $E(t)$ and $C(t)$. We then systematically exploit them in the following sections. Further discussion of the modelling of the global carbon cycle is found in appendix B.

A half-century of measurement of $C(t)$ and $E(t)$ shows that the annual increase of $C(t)$ is roughly half of the annual emissions $E(t)$. In other words, roughly half of the carbon emissions into the atmosphere stays in the atmosphere for a long time, while the other half is rapidly removed—in fact, by incorporation into the biosphere and the surface ocean. This empirical correlation can be approximately represented by

$$\frac{\mathrm{d}C}{\mathrm{d}t} \cong k \cdot E(t), \tag{2.1}$$

where k is a constant. We call equation (2.1) the *Constant Airborne Fraction Model*, because the airborne fraction is the fraction found by dividing the increase of CO_2 in the atmosphere by the fossil fuel CO_2 emissions in the same time period. The Constant Airborne Fraction Model subsumes all our ignorance about the details of the carbon cycle into the single factor, k. Combined with a definition of $E(t)$ that includes only fossil fuel emissions (and not, e.g. deforestation), this model separates biosphere issues from fossil-carbon issues.

Justification for the choice of a constant value of k, and for k somewhere near one-half, can be found in figure 1, which plots the airborne fraction over the 46-year-period from 1958 to 2004. On average, the airborne fraction is 58%. The remarkable year-to-year variability of the airborne fraction is discussed in

appendix A. To the nearest integer, today's emissions rate, E_0, is 8 GtC yr^{-1}, and the carbon content of the atmosphere is currently growing at 4 GtC yr^{-1}. This motivates us, for simplicity, whenever we use the Constant Airborne Fraction Model to obtain numerical results, to choose $k=0.5$. But the k dependence of all the derived relations is given explicitly, so that users of these relations can choose other values of k.

Many computer simulations of multi-century science-based global carbon cycle models have shown that small but finite and positive fossil-fuel emissions are allowed long after $C(t)$ has reached any designated stabilization value for the quantity of carbon in the atmosphere, C_{stab}. We denote by E_{stab} the value of $E(t)$ associated with $C(t)$ stabilized at C_{stab}. We have extracted the following empirical correlation between E_{stab} and C_{stab} from the global carbon cycle literature (Wigley *et al.* 1996)

$$E_{\mathrm{stab}} \approx \frac{C_{\mathrm{stab}} - 600}{200}\,\mathrm{GtC\ yr^{-1}}, \tag{2.2}$$

where C_{stab} is measured in GtC. The value of $E(t) - E_{\mathrm{stab}}$ at time t is a measure of the magnitude of the total emissions reduction effort needed to stabilize eventually at C_{stab}.

The Constant Airborne Fraction Model is not valid when $E(t)$ and E_{stab} are comparable, i.e. when $C(t)$ is stabilized or nearly stabilized. A generalization of equation (2.1) which is consistent with equation (2.2) is

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \lambda \cdot (E(t) - E_{\mathrm{stab}}), \tag{2.3}$$

where λ is chosen to enable equation (2.3) to approximate $\mathrm{d}C/\mathrm{d}t$ in the time domain of interest. If k is the value recommended for equation (2.1) at time t_0 by historical observational data, such as in figure 1, and $E(t)$ is expected to be moving towards E_{stab} for $t > t_0$, then $\lambda = k/[1-(E_{\mathrm{stab}}/E_0)]$ is a sensible choice for equation (2.3). We call equation (2.3) the *ad hoc* Model. For $C_{\mathrm{stab}} = 1000$ GtC, the model is: $\mathrm{d}C/\mathrm{d}t = 0.8 \cdot [E(t) - 3\ \mathrm{GtC\ yr^{-1}}]$, and for $C_{\mathrm{stab}} = 1200$ GtC, the model is $\mathrm{d}C/\mathrm{d}t = 0.67 \times [E(t) - 2\ \mathrm{GtC\ yr^{-1}}]$.

We will use both the Constant Airborne Fraction Model and the *ad hoc* Model, in the developments in this paper. We present an improved version of both of these models, a 'one-tank' model, in §7. We provide additional exposition on global carbon-cycle modelling in appendix B.

3. Constant pace mitigation

For any chosen C_{stab} higher than the current $C(t)$, the amount of *additional* atmospheric carbon content we can add to the atmosphere in the future is called the *headroom*:

$$H(t) \equiv C_{\mathrm{stab}} - C(t). \tag{3.1}$$

When someday $H(t)$ drops to zero, it means $C(t)$ has reached C_{stab}. There are two possibilities. One is that $H(t)$ then goes negative, signalling the breaching of C_{stab}. The other is that $H(t)$ goes to zero, but does not cross it. In the latter case, $E(t)$ manages to decrease from the current high value $E(t_0)$ to the much lower value E_{stab} estimated by equation (2.2), and stays thereafter at E_{stab}—which may be a slowly

decreasing function of time. Obviously, a monotonic $E(t)$ trajectory—once it commences the decrease—is highly desirable. Going below E_{stab} is problematic, because, as we discuss further in §7, it is already a small number.

Carbon emissions into the atmosphere from fossil fuels increase the value of $C(t)$. The time derivative of $C(t)$ can be related to its annual increment

$$\frac{\mathrm{d}C}{\mathrm{d}t}\cdot(1\ \mathrm{year}) = C(t) - C(t-1), \tag{3.2}$$

where time $(t-1)$ is one year before time t. This increase in the carbon content of the atmosphere is equal to the loss of headroom available to the world, assuming the world has not changed its target concentration during that year. Carbon mitigation means executing strategies to prevent the breaching of the chosen target level (i.e. to prevent the headroom from going negative). Future annual fossil-fuel carbon emissions into the atmosphere must be substantially reduced, relative to annual emissions today, in order for $\mathrm{d}C/\mathrm{d}t$ to go to zero, in spite of the anticipated increases in world energy demand and population in the next few decades.

We choose the *Constant Pace Mitigation strategy*, to be referred to in the rest of the paper as the CPM strategy,[2] as our benchmark mitigation strategy. The CPM strategy requires $\mathrm{d}C/\mathrm{d}t$ to start its decline towards zero *immediately* (at $t = t_0$) and to fall *at a constant pace*, such that $C(t)$ reaches C_{stab} at the same year as $\mathrm{d}C/\mathrm{d}t$ reaches zero. With either the Constant Airborne Fraction or the *ad hoc* Model, the annual emissions reduction also proceeds at a constant pace. Figure 2 provides a geometric representation of the strategy. It can be shown that the constant pace of the CPM strategy is smaller than the maximum pace of every other possible strategy.

A constant-pace trajectory means the $C(t)$ curve to the right of $t = t_0$ in figure 2*a* is a parabola which is tangent to the historical $C(t)$ data at time t_0 and also tangent to the horizontal line at C_{stab}. It reaches the C_{stab} line at time $t_0 + \tau_{\mathrm{CPM}}$. The associated $\mathrm{d}C/\mathrm{d}t$ trajectory in figure 2*b* is a straight line, and the area under this line is the headroom $H(t_0)$. Using only the formula for the area of a triangle, we can easily obtain

$$H(t_0) = \frac{\tau_{\mathrm{CPM}}(t_0)}{2}\left(\frac{\mathrm{d}C}{\mathrm{d}t}\right)_0 \quad \mathrm{GtC}. \tag{3.3}$$

We can solve equation (3.3) for $\tau_{\mathrm{CPM}}(t_0)$

$$\tau_{\mathrm{CPM}}(t_0) = \frac{2\cdot[C_{\mathrm{stab}} - C(t_0)]}{C(t_0) - C(t_0 - 1)} \quad \mathrm{year}. \tag{3.4}$$

Note that $\tau_{\mathrm{CPM}}(t)$ as given by equation (3.4) can be computed at any time for any chosen value of C_{stab} using only observational $C(t)$ data. This number so computed is independent of k and the veracity of equation (2.2), and no modelling parameters at all are involved. The parameter, $\tau_{\mathrm{CPM}}(t)$, is the characteristic time-scale for constant-pace mitigation; it is measured in years. We will make extensive use of $\tau_{\mathrm{CPM}}(t)$ in this paper.

We define the *pace* of Constant Pace Mitigation (CPM) strategy, $P_{\mathrm{CPM}}(t)$, as the average rate at which the emissions rate, $E(t)$, is reduced to reach the value, E_{stab}, after a time of exactly $\tau_{\mathrm{CPM}}(t)$

$$P_{\mathrm{CPM}}(t) \equiv \frac{E(t) - E_{\mathrm{stab}}}{\tau_{\mathrm{CPM}}(t)}\ \mathrm{GtC\ yr^{-1}\ per\ year}. \tag{3.5}$$

[2] See appendix C for a full list of symbols, subscripts, superscripts, and acronyms.

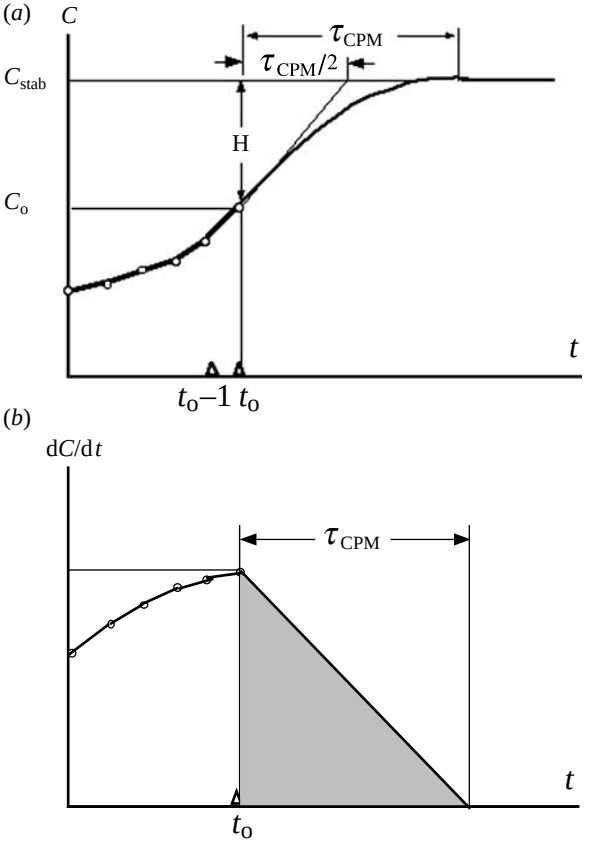

Figure 2. Two geometric representations of the definition of the time-scale for constant-pace mitigation, τ_{CPM}. (*a*) $C(t)$ versus time, and (*b*) dC/dt versus time. The definition of τ_{CPM} requires only observational $C(t)$ data and a particular C_{stab} target level.

Note that both equations (3.4) and (3.5) are independent of carbon cycle modelling when observational data for $E(t)$ and $C(t)$ are available and equation (2.2) is used to estimate E_{stab}.

How does $E(t)$ influence future values of $\tau_{CPM}(t)$, for which, of course, we do not have access to observational data? We then need carbon cycle models. We can use the *ad hoc* Model to eliminate $C(t)-C(t-1)$ from equation (3.4), with the help of equations (2.3) and (3.2), in favour of $\lambda\cdot[E(t)-E_{stab}]$. We can now relate the CPM time-scale, $\tau_{CPM}(t)$, directly to $E(t)$

$$\tau_{CPM}(t) \cong \frac{2\cdot(C_{stab}-C(t))}{\lambda\cdot[E(t)-E_{stab}]} \quad \text{year}. \tag{3.6}$$

For simplicity, and especially if the future $E(t)$ trajectory of interest is well above E_{stab} and moving away, the Constant Airborne Fraction Model can be used instead, i.e. the denominator in equation (3.6) can be replaced by $k\cdot E(t)$.

For either the Constant Airborne Fraction Model, equation (2.1), or the *ad hoc* Model, equation (2.3), the CPM $E(t)$ trajectory is linear with time. Note, however, that our definitions of $\tau_{CPM}(t)$ and $P_{CPM}(t)$ allow both variables to be

computed along *any* emissions trajectory and, therefore, as we will see in §6, can be used to monitor the performance of *any* ongoing mitigation strategy.

As mentioned previously, the CPM strategy has a special property, relative to all other mitigation strategies. For the same headroom, all other trajectories must have a higher pace and a lower pace at least once, relative to the constant pace of the CPM strategy adopted immediately. A slower constant pace than the CPM pace would overshoot the target C_{stab} before falling back, while a faster constant pace would earn a slower approach to the same C_{stab}. Pace, being the amount of reduction per year in $E(t)$, is a quantitative measure of mitigation effort. Thus, among all possible strategies for a fixed value of E_{stab}, the CPM strategy is of special interest, because it has the lowest maximum annual effort in comparison to all others that honour the chosen C_{stab}.

(*a*) *Some numbers*

The current value of $C(t)$ is approximately 800 GtC, and it is increasing by approximately 4 GtC each year. The latter number is consistent with the current $E(t)$ of roughly 8 GtC yr^{-1} and the Constant Airborne Fraction Model.[3] The current $E(t)$ is increasing by approximately 0.16 GtC yr^{-1} per year. In the thousand years before 1800, the carbon content of the atmosphere varied by less than 4%, according to measurements of bubbles trapped in polar ice (Siegenthaler *et al.* 1988; cited in Sarmiento *et al.* 1992). The average pre-industrial concentration level is usually quoted as 280 p.p.m., which converts, using the Rosetta stone, to approximately 600 GtC. We denote this quantity to be C_{pre}. We denote by C_{2x} twice this pre-industrial value, or an atmospheric carbon content of 1200 GtC. Since the current value of $C(t)$ is very close to 800 GtC, we are already one-third of the way towards C_{2x}.

We explore two values of C_{stab} which bracket the range of stabilization targets discussed by the environmental science community, activist citizens, and many political leaders.

(i) $C_{\mathrm{stab}} = C_{2x} = 1200$ GtC (560 p.p.m.). This is the doubling target, which is the most frequently discussed target. For this case, E_{stab} is approximately 3 GtC yr^{-1}, and the current headroom is $H(t) = 400$ GtC. From equation (3.4), the current mitigation time is $\tau_{\mathrm{CPM}}(t) = 200$ years, and, from equation (3.5), the average pace of annual emission reduction is $P_{\mathrm{CPM}}(t) = 0.025$ GtC yr^{-1} per year. Thus, if we immediately start the CPM strategy, $\tau_{\mathrm{CPM}}(t)$ for $C_{\mathrm{stab}} = C_{2x}$ is a multi-century number. The needed total emissions reduction is roughly two-thirds of the current emissions.

(ii) $C_{\mathrm{stab}} = 1000$ GtC (470 p.p.m.). This is a much more stringent target, which is substantially below C_{2x}. This target is advocated by many environmental scientists at this time. For this case, E_{stab} is approximately 2 GtC yr^{-1}, the current headroom is $H(t) = 200$ GtC, the current constant-pace mitigation time is $\tau_{\mathrm{CPM}} = 100$ years, and the required average pace of annual emission reduction is $P_{\mathrm{CPM}}(t) = 0.06$ GtC yr^{-1}.

[3] In the wedge papers of Pacala and Socolow (Pacala & Socolow 2004; Socolow & Pacala 2006) global carbon emissions from fossil fuels and cement were reported at 7 GtC yr^{-1}, which led, via further assumptions, to 'seven wedges'. The emissions rate in the data base we are using was 7.3 GtC yr^{-1} in 2003 Marland *et al.* 2006) and is surely closer to 8 GtC yr^{-1} than to 7 GtC yr^{-1} today.

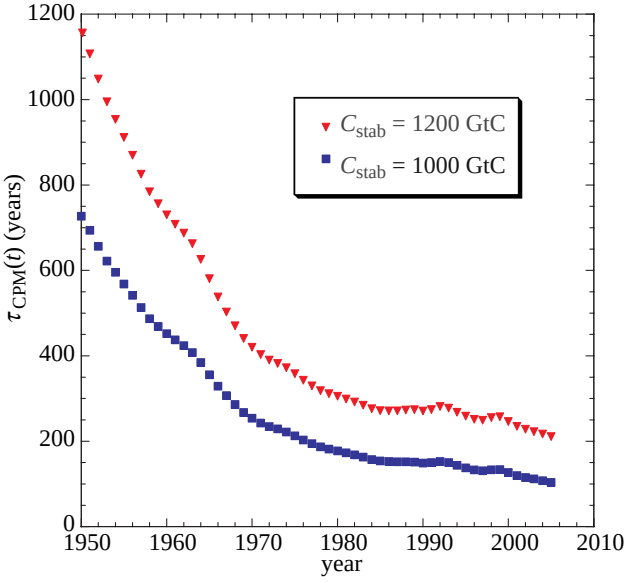

Figure 3. Sixty years of historical data for $\tau_{\mathrm{CPM}}(t)$, the time required to reach C_{stab} along a Constant Pace Mitigation trajectory, for C_{stab}=1200 GtC and C_{stab}=1000.

In figure 3, we show the historical values of τ_{CPM} from 1950 to 2006, for our two target concentrations. We see that τ_{CPM} for stabilization at C_{2x} was about 410 years around 1970 about 36 years ago, and it has decreased to about 200 years, or by 6 years per year on the average. It is, of course, understood that whenever $\tau_{\mathrm{CPM}}(t)$ computed from equation (3.4) crosses zero, the chosen value of C_{stab} is being breached. Thus, the trend of τ_{CPM} is very informative.

From figure 3, we see that if we start the CPM strategy right now, the $\tau_{\mathrm{CPM}}(t)$'s for both above cases are century-scale numbers. We suspect that many readers are surprised that the current values of τ_{CPM}'s are so large. We learn from figure 3 that the global warming problem is a problem that can be addressed gradually, over a period extending more than a century, if we start CPM immediately. As we will see in §6, procrastination shortens τ_{CPM} and increases $\mathrm{P}_{\mathrm{CPM}}$.

4. Simplifications when $E(t) \gg E_{\mathrm{stab}}$

Equation (3.5) is the definition of the constant-pace-mitigation pace, P_{CPM}. Eliminating τ_{CPM} from equation (3.5) using equation (3.4) with the *ad hoc* Model, we obtain

$$P_{\mathrm{CPM}}(t) = \frac{\lambda \cdot (E(t) - E_{\mathrm{stab}})^2}{2 \cdot [C_{\mathrm{stab}} - C(t)]}. \tag{4.1}$$

The presence of the E_{stab} in equation (4.1) makes it awkward to use. Instead if the Constant Airborne Fraction Model is used in equation (4.1), we have

$$P_{\mathrm{CPM}}(t) \cong \frac{k \cdot [E(t)]^2}{2 \cdot [C_{\mathrm{stab}} - C(t)]}. \tag{4.2}$$

Equation (4.2) is much more convenient for back-of-the envelope calculations. It is obviously not a valid formula to use when the global warming crisis is nearly over because $E(t)$ is then near E_{stab}.

5. What C_{stab} is achievable if we start now and assume there is a limit on how hard we can work?

A low C_{stab} is obviously desirable. What is the lowest possible C_{stab} that we can achieve if we immediately begin a CPM strategy with the largest possible pace of reduction of annual emissions, $P_{\mathrm{CPM}}^{\max}$, and continue reducing emissions at that pace for all future years until the job is done? A maximum possible pace might be inferred, for example, from an evaluation of social dislocation and year-to-year mitigation cost. Substituting $P_{\mathrm{CPM}}^{\max}$ for P_{CPM} in equation (4.1) and solving for $C_{\mathrm{stab}}^{\min} - C(t)$, we obtain

$$C_{\mathrm{stab}}^{\min} - C(t) = \frac{\lambda \cdot [E(t) - E_{\mathrm{stab}}]^2}{2P_{\mathrm{CPM}}^{\max}}. \tag{5.1}$$

We see that the achievable headroom is proportional to the square of the current value of $E(t) - E_{\mathrm{stab}}$ and inversely proportional to the value of $P_{\mathrm{CPM}}^{\max}$ immediately adopted and held constant until the job is done.

The steeper the CPM pace, the more difficult is the mitigation effort. Because the world does not yet have any experience with the deliberate reduction of global CO_2 emissions, we can only guess at the plausible range of values for $P_{\mathrm{CPM}}^{\max}$. We explore several values of $P_{\mathrm{CPM}}^{\max}$ in §6.

6. The penalties of procrastination

The global CO_2 emissions rate is currently going up, not down. We do not know how much time will pass before the emissions rate starts to fall, nor do we know the form of the transition from rising emissions to falling emissions. In this section, we consider in some detail three kinds of transition to stabilization and then provide a general tool for the year-to-year assessment of any trajectory.

(*a*) *Business as usual, then begin constant pace mitigation immediately*

We assume an emissions trajectory with two time periods. In the first period, emissions grow at a constant pace, and in the second period, they fall at a constant pace. The first period is called Business As Usual (BAU), and the second period is the period of CPM.

For the BAU period, we assume that the annual increase of $E(t)$ is $0.16\ \mathrm{GtC\ yr^{-1}}$ per year. A half century of BAU starting in 2006 would then bring $E(t)$ in 2056 to $16\ \mathrm{GtC\ yr^{-1}}$, double the current emissions rate; a doubling of the emissions rate over 50 years is an approximation to many estimates of emissions growth in a world with minimal concern for carbon mitigation (Pacala & Socolow 2004). The Constant Airborne Fraction Model is the model of choice, because $E(t)$ is considerably larger than E_{stab}. If the BAU phase lasted for 50 years, this would result in 600 GtC of cumulative emissions and, assuming $k = 0.5$, would use up 300 GtC of the current headroom.

Table 1. Constant Pace Mitigation (CPM) begins immediately after a period of BAU during which $E(t)$ increases at 0.16 GtC yr^{-1} per year. The emissions rate and CO_2 level at the moment of transition are given in the second and third columns. The remaining columns show the parameters, τ_{CPM} and P_{CPM}, during the mitigation period, for our two values of C_{stab}. Those cases where P_{CPM} exceeds 0.16 GtC yr^{-1} per year are in bold italics (see text). In all calculations, $P_{CPM}(t_0)$ is computed using equation (4.2) and τ_{CPM} is computed using $\tau_{CPM}(t_0) = 2\cdot[C_{stab} - C(t_0)]/[k\cdot E(t_0)]$, with $k=0.5$ in both cases; t_0 is the moment of transition.

CPM starts (yr)	emissions $E(t)$(GtC yr^{-1})	CO_2 level $C(t)$ (GtC)	$C_{stab}=1000$ GtC		$C_{stab}=1200$ GtC	
			τ_{CPM} (yr)	P_{CPM} (GtC yr^{-2})	τ_{CPM} (yr)	P_{CPM} (GtC yr^{-2})
2006	8.0	800	100	0.08	200	0.04
2016	9.6	844	65	0.15	148	0.06
2026	11.2	896	37	***0.30***	109	0.10
2036	12.8	956	14	***0.93***	76	***0.17***
2046	14.4	1024	—	—	49	***0.29***
2056	16.0	1100	—	—	25	***0.64***

For the pace of descent of the emissions rate, P_{CPM}^{max}, we study three values: 0.04, 0.08 and 0.16 GtC yr^{-1} per year. These values are 0.5, 1 and 2% per year, respectively, of 8 GtC yr^{-1}, the current E_0 value. The largest of the three values, 0.16 GtC yr^{-1} per year, is equal in *magnitude* to the rate of $E(t)$ *increase* assumed in the BAU period, and the other two values are one quarter as steep and half as steep.

To get a feel for the magnitude of these mitigation efforts, we adapt the concept of the Pacala–Socolow wedge (Pacala & Socolow 2004). In the Pacala–Socolow paper, a one-wedge effort is an effort that reduces the emissions rate, $E(t)$, by 1 GtC yr^{-1} in 50 years, or 0.02 GtC yr^{-1} per year, and wedges reduce the rising $E(t)$ that is expected from BAU. Here, a one-wedge effort is simply a reduction in the emissions rate over *one year* by 0.02 GtC yr^{-1} per year

one-wedge effort = reduction of the emissions rate by 0.02 GtC yr^{-1} in one year.

Then, $P_{CPM}=0.04$ GtC yr^{-1} per year can be called a 'two-wedge' effort. If a two-wedge CPM effort were to start now, it would bring the emissions rate to zero in 200 years. To be sure, since the stabilized emission rate is finite and positive, the emissions rate does not have to be brought all the way to zero to achieve stabilization.

We can equate a one-wedge effort to a rate at which conventional coal plants are decommissioned and replaced with non-fossil power plants (or with coal plants that capture and store CO_2). A typical 1 GW base-load coal plant operating today will emit about 2 million tons of carbon per year. Closing down 10 such plants in a given year and commissioning an equivalent amount of non-CO_2-emitting power is a one-wedge effort that year. That effort will be undone if there is a net addition to fossil-carbon plants elsewhere in the same system, so we must also assume that no such net addition occurs. The year that Three Gorges Dam was commissioned, with its power output of nearly 20 GW, would have been a two-wedge effort that year, if our other conditions had also been met. Sustaining a two-wedge effort would require commissioning a Three Gorges Dam *and* meeting the other conditions *every year*. All *growth* in output of conventional coal power represents negative-wedge effort.

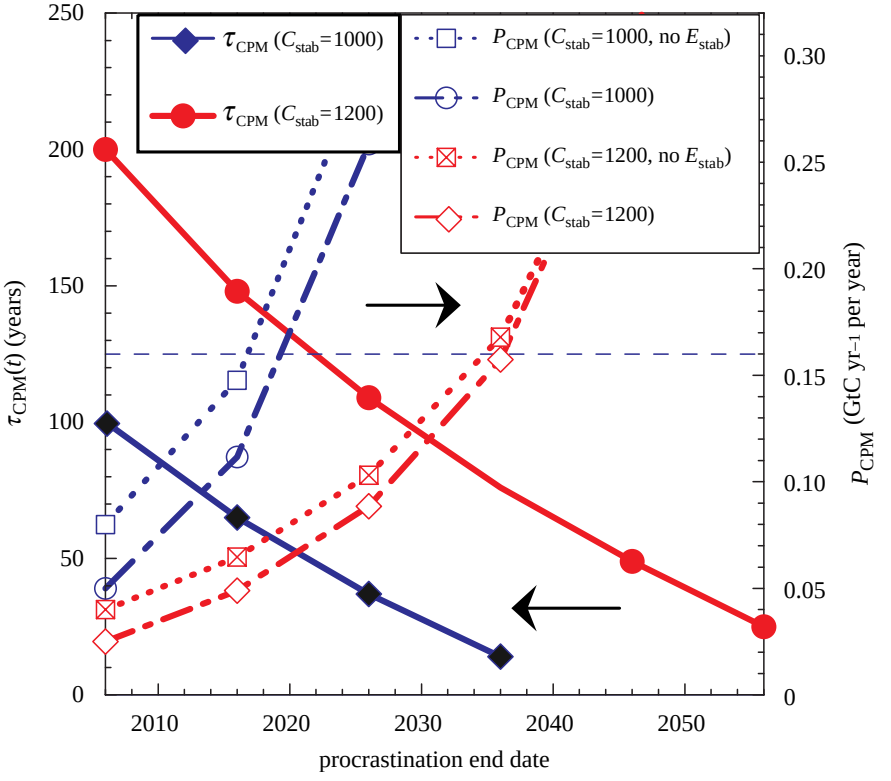

Figure 4. Throughout the period of procrastination (BAU) $E(t)$ rises at 0.16 GtC yr^{-1} per year; then, when procrastination ends, Constant Pace Mitigation begins immediately. The later the end-date for the procrastination and the lower the specified value of C_{stab}, the shorter the time for constant-pace mitigation, $\tau_{\mathrm{CPM}}(t)$, and the faster the pace required for constant-pace mitigation, $P_{\mathrm{CPM}}(t)$. Pairs of curves for P_{CPM} are shown: the dash-dot lines use equation (3.6) to calculate $P_{\mathrm{CPM}}(t)$, with E_{stab} calculated from equation (2.2), while the dotted lines use the variant of equation (3.6) with $E_{\mathrm{stab}}=0$. (See text for more explanation.) The specific value of the pace singled out by a thin dashed horizontal line, 0.16 GtC yr^{-1} per year, is the value of the pace which, when exceeded, results in bold italic entries in table 1.

Table 1 gives the values of τ_{CPM} and P_{CPM}, for one to five decades of BAU, followed by CPM, for both C_{stab} target levels (1000 and 1200 GtC). Because E is moving away from E_{stab}, we use the Constant Airborne Fraction Model, so $P_{\mathrm{CPM}}(t_0)$ is computed using equation (4.2) and $\tau_{\mathrm{CPM}}(t_0)$ is computed using $2\cdot[C_{\mathrm{stab}}-C(t_0)]/[k\cdot E(t_0)]$, with $k=0.5$ in both cases. Here, t_0 is the end of the procrastination period. For example, following BAU for 30 years would shrink τ_{CPM} for the 1200 GtC target to 76 years (recall that today it is 200 years); it would shrink the τ_{CPM} for the 1000 GtC target to 14 years (today it is 100 years).

Values of P_{CPM} in excess of 0.16 GtC yr^{-1} per year might be judged to be excessively difficult. In bold italics in table 1 are those cases where P_{CPM} exceeds this value. The 1000 GtC target is inaccessible in this sense soon after 2016 and the 1200 GtC target is inaccessible shortly before 2036. From this perspective, procrastination transforms climate change mitigation from a problem that can be addressed gradually, over a century or more into a problem extending only over a few decades (Mignone *et al.* submitted).

Figure 4 graphs the same relationships as table 1. As in table 1, τ_{CPM} is calculated using $\tau_{\mathrm{CPM}}(t_0) = 2\cdot[C_{\mathrm{stab}} - C(t_0)]/[k\cdot E(t_0)]$, because we are moving away from E_{stab}, and the Constant Airborne Fraction Model is assumed to be valid at t_0. Figure 4 shows *two* curves for P_{CPM}. The dash-dot curve is calculated from equation (3.5); we use equation (2.2) to estimate E_{stab} values of 2 and 3 GtC yr^{-1}, for $C_{\mathrm{stab}}=1000$ and 1200 GtC, respectively. These curves are consistent, assuming that the end point of stabilization will require $E=E_{\mathrm{stab}}$. The dotted curve is calculated using the variant of equation (3.5) where $E_{\mathrm{stab}}=0$; it is valid only with the simplifying assumption that the total job, E_0-E_{stab}, can be approximated by E_0. The ratio of the two P_{CPM} curves is, therefore, $E(t_0)/[E(t_0)-E_{\mathrm{stab}}]$.

Equation (5.1) is the tool to use when there is a fastest possible pace. Figure 5 displays the minimum achievable C_{stab} for three values of this maximum pace, initiated immediately after a specified period of BAU. During the BAU period, $E(t)$ rises at 0.16 GtC yr^{-1} per year, and during the CPM period the pace, P_{CPM}, takes on our three values: 0.04 GtC yr^{-1} per year (a 'two-wedge effort'), 0.08 GtC yr^{-1} per year (a 'four-wedge effort'), and 0.16 GtC yr^{-1} per year (an 'eight-wedge effort'). Figure 5 also assumes the Constant Airborne Fraction Model.

For example, if, after 50 years of procrastination, P_{CPM} is fixed at 0.16 GtC yr^{-1} per year, we see from figure 5 that we can stabilize at no lower than $C_{\mathrm{stab}}=1500$ GtC. We understand this value as follows. During the 50 years of procrastination, 600 GtC are emitted to the atmosphere, and, with $k=0.5$, 300 GtC stay in, leading to an atmosphere containing 1100 GtC (see table 1). Neglecting E_{stab} in comparison to $E(t_0)$, a descent to zero from an emissions rate of 16 GtC yr^{-1} will have a pace of 0.16 GtC yr^{-1} per year if it extends over $\tau_{\mathrm{CPM}}=100$ years, i.e. from 2056 to 2156. In that century, another 800 GtC are emitted and another 400 GtC are added to the atmosphere, leading to $C_{\mathrm{stab}}=1500$ GtC.

Similarly, suppose we believe that $P^{\max}_{\mathrm{CPM}} = 0.16$ GtC yr^{-1} per year is the fastest pace we are able to sustain on a CPM trajectory, and we use equation (4.2) for the CPM period, thereby neglecting E_{stab} relative to $E(t)$. It follows that for a $C_{\mathrm{stab}}=C_{2x}=1200$ GtC target, we must end BAU after 29 years, at a peak emissions rate of 12.6 GtC yr^{-1}. For a 1000 GtC target, we must end BAU after 11 years at an emissions rate of 9.8 GtC yr^{-1}. These two trajectories are plotted in figure 6*a*,*b*.

Figure 5 allows us to bracket the date at which we must stop procrastinating. If our goal is to honour $C_{\mathrm{stab}}=1000$ GtC, we can procrastinate for about a decade with $P^{\max}_{\mathrm{CPM}} = 0.16$ GtC yr^{-1} per year, but must begin immediately if $P^{\max}_{\mathrm{CPM}}$ is 0.08 GtC yr^{-1} per year. If $P^{\max}_{\mathrm{CPM}}$ is only 0.04 GtC yr^{-1} per year (we can at most marshal a two-wedge effort), this target is already inaccessible. If our goal is to honour $C_{\mathrm{stab}}=1200$ GtC, we can procrastinate for about three decades if we can achieve $P^{\max}_{\mathrm{CPM}}$ at 0.16 GtC yr^{-1} per year and for *ca* 15 years if we can achieve at most $P^{\max}_{\mathrm{CPM}}$ at 0.08 GtC yr^{-1} per year. We must begin constant-pace mitigation immediately if our best $P^{\max}_{\mathrm{CPM}}$ is 0.04 GtC yr^{-1} per year.

(*b*) *Neither up nor down for a while; just keep emissions flat, then start CPM*

In a 2004 paper, one of us (R.H.S.) and Pacala proposed a schedule of CO_2 emissions in two parts: for 50 years, emissions are flat, and thereafter they fall (Pacala & Socolow 2004). Here, we consider a related, general class of emissions trajectories, where emissions are held constant for an arbitrary period (i.e. not

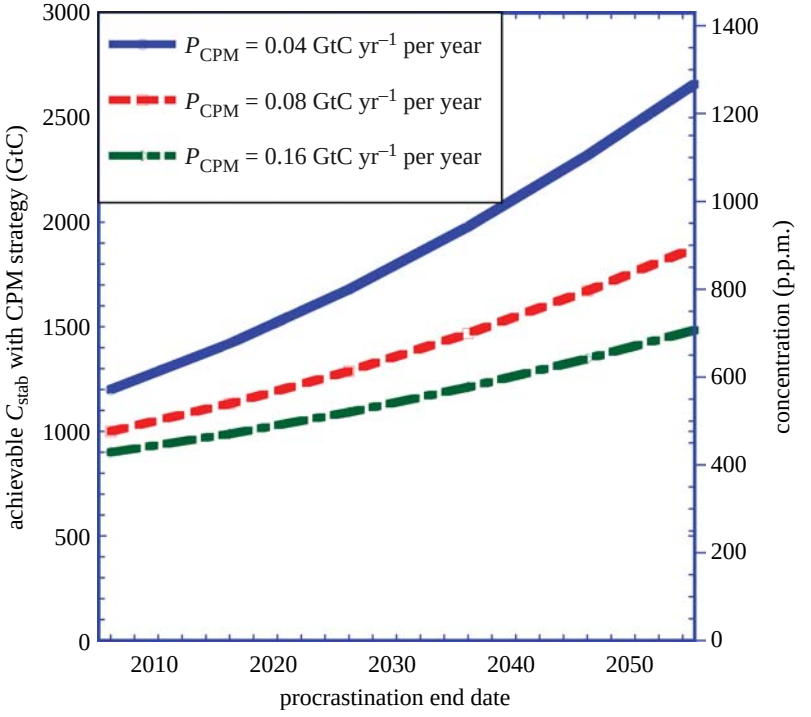

Figure 5. Achievable C_{stab} if we start the CPM strategy immediately after a period of BAU, during which $E(t)$ rises at 0.16 GtC yr^{-1} per year. Constant-pace mitigation is assumed throughout the CPM period, with three values of P_{CPM}: 0.04 GtC yr^{-1} per year (a 'two-wedge effort,' the solid line); 0.08 GtC yr^{-1} per year (a 'four-wedge effort', the dashed line); and 0.16 GtC yr^{-1} per year (an 'eight-wedge effort', the dash-dot line). The constant airborne fraction model is used, with k=0.5.

necessarily 50 years), after which, immediately, there is a CPM period. This class of trajectories is particularly simple to analyse using either equation (4.1) or (4.2), both of which say that the product $H(t)\cdot P_{\mathrm{CPM}}(t)$ does not change with time during the period of constant E (with E_{stab} estimated by equation (2.2), if needed). As $C(t)$ continues to rise with E fixed, the values of $H(t)$ and τ_{CPM} must decrease and the value of $P_{\mathrm{CPM}}(t)$ must increase.

We shall for the sake of simplicity use equation (4.2), with k=0.5. When emissions are held constant at 8 GtC yr^{-1}, we have: $H(t)\cdot P_{\mathrm{CPM}}(t) = 16$ GtC^2/yr^2. The Constant Airborne Fraction Model can be used to compute how the headroom, $H(t)$, falls during the period of constant emissions. For example, by 2056, 400 GtC are emitted and, if k=0.5, 200 GtC are added to the atmosphere. The 1000 GtC (470 p.p.m.) stabilization target becomes inaccessible at 2056—we have no more headroom. But the 1200 GtC (560 p.p.m.) stabilization target, C_{2x}, still has 200 GtC of headroom in 2056. Equation (4.2) tells us that constant-pace mitigation starting in 2056 will use up this headroom with P_{CPM} constant at 0.08 GtC yr^{-1} per year (a four-wedge effort). The value of τ_{CPM} has now dropped to 100 years, so the CPM trajectory will bring the carbon content of the atmosphere to its 1200 GtC stabilization value in 2156.

If we have decided in advance that the CPM period is governed by a maximum pace, $P^{\max}_{\mathrm{CPM}}$, then, for $E_0 = 8$ GtC yr^{-1}, the headroom at the time of onset of the CPM period, t_0, will be found from $H(t_0)\cdot P^{\max}_{\mathrm{CPM}}(t) = 16$ GtC^2 per yr^2. For example, if $P^{\max}_{\mathrm{CPM}} = 0.16$ GtC yr^{-1} per year, the headroom at the onset of the mitigation period must drop to 100 GtC. For example, if $C_{\mathrm{stab}} = 1200$ GtC yr^{-1}, 300 GtC of the initial 400 GtC of headroom are used up during the constant-emissions period. If the Constant Airborne Fraction Model is used for the constant-emissions period, with $k = 0.5$, then the cumulative emissions during the constant-emissions period are 600 GtC, which therefore lasts $600/8 = 75$ years. By contrast, if $C_{\mathrm{stab}} = 1000$ years, the initial headroom is 200 GtC, and the constant-emissions period lasts only 25 years, using up 100 GtC of headroom with 200 GtC of cumulative emissions. In both cases, the duration of the follow-on CPM period is $2\cdot H(t_0)/(k\cdot E_0) = 50$ years. These two trajectories are plotted on figure 6*a*,*b*.

(*c*) *Parabolic mitigation instead of CPM*

Since a significant amount of effort is required to alleviate the rising BAU $E(t)$ trajectory, mitigation strategies that have continuous $E(t)$ and $\mathrm{d}E/\mathrm{d}t$ are obviously preferable. Thus a typical $E(t)$ trajectory would start with E_0, rise for a while, and then drop towards the much lower value of E_{stab}, reaching it at the same year $C(t)$ arrives at the chosen target level, C_{stab}.

The simplest $E(t)$ trajectory that has a continuous $\mathrm{d}E/\mathrm{d}t$ at the start would be a *parabolic mitigation* trajectory. We denote the total amount of time of this mitigation strategy by $\tau_{\mathrm{PM}}(t_0)$, and introduce $\eta = (t - t_0)/\tau_{\mathrm{PM}}(t_0)$, so that η runs from 0 to 1. The parabolic $E(t)$ trajectory can now be written as follows:

$$E(t) = E_{\mathrm{stab}} + (E_0 - E_{\mathrm{stab}})(1 + s\eta - (1 + s)\eta^2), \tag{6.1}$$

where s is a dimensionless parameter representing certain initial conditions of $E(t)$ at $t = t_0$

$$s \equiv \frac{\tau_{\mathrm{PM}}(t_0)(\mathrm{d}E/\mathrm{d}t)_0}{E_0 - E_{\mathrm{stab}}}. \tag{6.2}$$

With a positive s, $E(\eta)$ initially rises, peaks at $\eta = s/(2(1 + s))$, and decreases to E_{stab} at $\eta = 1$. The emissions rate at the peak is $E_{\mathrm{stab}} + (E_0 - E_{\mathrm{stab}})(1+s/2)^2/(1+s)$. The headroom, H_0, is the integral of $\mathrm{d}C/\mathrm{d}t$ from $t = t_0$ to $t = t_0 + t_{\mathrm{PM}}$, and, for the *ad hoc* Model, $\mathrm{d}C/\mathrm{d}t$ is given by equation (2.3). Integrating equation (6.1) from $\eta = 0$ to 1, and assuming the *ad hoc* Model, we obtain

$$H_0 = \frac{(s + 4)}{6}\tau_{\mathrm{PM}}(t_0)\left(\frac{\mathrm{d}C}{\mathrm{d}t}\right)_0. \tag{6.3}$$

Using equation (3.4), we can relate $\tau_{\mathrm{PM}}(t_0)$ to $\tau_{\mathrm{CPM}}(t_0)$

$$\tau_{\mathrm{PM}}(t_0) = \frac{3\tau_{\mathrm{CPM}}(t_0)}{s + 4}. \tag{6.4}$$

Using this in equation (6.2), we obtain a quadratic equation for s. Solving this quadratic equation for s, we obtain

$$s = \sqrt{4 + \frac{3(\mathrm{d}E/\mathrm{d}t)_0}{P_{\mathrm{CPM}}(t_0)}} - 2, \tag{6.5}$$

where we use the *ad hoc* Model, equation (2.3), to relate $\mathrm{d}C/\mathrm{d}t$ to $E(t)$.

The analogues of equations (6.1) and (6.2) for the Constant Airborne Fraction Model are obtained by setting $E_{\mathrm{stab}}=0$. Equations (6.3), (6.4) and (6.5) are unchanged.

Because of the symmetry of the parabola about the emission peak, the magnitude of the *downward* slope of the parabola will pass through the same value as the magnitude of the initial upward slope at an intermediate time, when the emissions rate passes downward through the value of the initial emissions rate, E_0. Thereafter, the downward slope keeps increasing in magnitude, and in the final year, $t=t_0+\tau_{\mathrm{PM}}$, it reaches its maximum magnitude. The ratio of this maximum downward slope to the constant pace P_{CPM} of the CPM strategy is $(s+2)\cdot(s+4)/3$.

We can estimate the parameters of the mitigation parabola, if parabolic mitigation were to begin today and the goal is to cap the carbon content of the atmosphere at twice its preindustrial concentration. Then $C_{\mathrm{stab}}=1200$ GtC and today's headroom, H_0, is 400 GtC. We assume that today's emissions rate, E_0, is 8 GtC yr^{-1} and that its rate of change today, $(\mathrm{d}E/\mathrm{d}t)_0$, is 0.16 GtC yr^{-1} per year. We assume that $\mathrm{d}C/\mathrm{d}t$ today is 4 GtC yr^{-1}, so τ_{CPM} is 200 years. We assume for simplicity that E_{stab} can be neglected in comparison to $E(t)$, so that $P_{\mathrm{CPM}}=E_0/\tau_{\mathrm{CPM}}$ [the $E_{\mathrm{stab}}=0$ limit of equation (3.5)], and thus P_{CPM} is 0.04 GtC yr^{-1} per year. The second term under the square root in equation (6.5) is 12, and s is exactly 2. From equation (6.2), parabolic mitigation requires $\tau_{\mathrm{PM}}=100$ years. The emissions rate peaks when $\eta=1/3$, i.e. $100/3=33$ years from now. From equation (6.1) with $s=2$, the peak emissions rate is $(4/3)\cdot E_0$, or 10.7 GtC yr^{-1}. The downward slope is 0.32 GtC yr^{-1} per year in the final year of the trajectory, twice as large in magnitude as the initial slope. The two parabolic trajectories just described are plotted in figure 6*a*,*b*.

Similarly, we find that $s=1.16$ when $C_{\mathrm{stab}}=1000$ GtC. The mitigation parabola rises to a peak emissions rate of 9.2 GtC yr^{-1} after 16 years, and parabolic mitigation is complete after $\tau_{\mathrm{PM}}=58$ years. The downward slope is 0.44 GtC yr^{-1} per year in the final year.

With any positive s, we are a long way from the CPM linear trajectory, which is identified with $s=-1$.

(*d*) *Summary of the stabilization trajectories*

In figure 6*a*,*b*, we display four emissions trajectories developed earlier, and in figure 6*c*,*d* we display their associated $C(t)$, all under the $E_{\mathrm{stab}}=0$ assumption. Hence, we use the Constant Airborne Fraction Model, equation (2.1), a crude but useful tool, to obtain their associated $C(t)$ trajectories and display them in figure 6*c*,*d*. Recall that the Constant Airborne Fraction Model approximates the rate of change of the carbon content of the atmosphere as a constant fraction of the emissions rate. As we have already warned the reader several times, this model overestimates the rate of change of the carbon content of the atmosphere when $E(t)$ is comparable to E_{stab}. When $E(t)$ is less than E_{stab} given by equation (2.2), even the sign of $\mathrm{d}C/\mathrm{d}t$ is wrong. Nonetheless, as long as this caveat regarding $E(t)$ not being comparable to E_{stab} is kept in mind, figure 6 allows a useful comparison across the mitigation strategies we have just been considering.

In both figure 6*a*,*b*, one of the four trajectories is a CPM trajectory where $E(t)$ begins to decrease immediately, as discussed in §3. The other three are examples of emissions trajectories discussed in §6*a*–*c*. Figure 6*a*,*c* are for $C_{\mathrm{stab}}=1000$ GtC

and figure 6*b*,*d* are for 1200 GtC. The areas under the four trajectories in figure 6*a*,*b* are the same and are equal to twice the initial headroom (because $k=0.5$): 400 and 800 GtC for the trajectories in figure 6*a*,*b*, respectively.

All the specific trajectories plotted here have been discussed explicitly in earlier sections. We arbitrarily constrain both the 'BAU, then CPM' $E(t)$ trajectory (§6.1) and the 'flat, then CPM' $E(t)$ trajectory (§6.2) to have the same $P_{\mathrm{CPM}}^{\mathrm{max}}$, 0.16 GtC yr^{-1} per year by adjusting the number of years before starting CPM; as a result, the downward portions of these two trajectories are parallel. The 'parabolic emissions' $E(t)$ trajectory and the 'BAU, then CPM' $E(t)$ trajectory are constrained to have the same initial *upward* slope, also 0.16 GtC yr^{-1} per year. We see from figure 6 that requiring the same headroom and either the same upward slope or the same downward slope constrains these three trajectories to be very close together.

To be sure, the vertical axis in both figure 6*a*,*c* can be rescaled to represent $\mathrm{d}C/\mathrm{d}t$, instead of $E(t)$, by dividing all the entries by 2: in either figure, whenever $E(t)$ is 10 GtC yr^{-1}, for example, $\mathrm{d}C/\mathrm{d}t$ is 5 GtC yr^{-1}. We are not using any carbon-cycle modelling in figure 6. We are simply assuming that the emissions rate is twice the rate of increase in atmospheric carbon.

Improved emissions trajectories relative to those shown in figure 6 are obtained if we use the *ad hoc* Model (equation (2.3)), where the rate of change of the carbon content of the atmosphere falls to zero when emissions fall to E_{stab}, given by equation (2.2). We briefly consider the four emissions trajectories motivated by the *ad hoc* Model, which correspond to each of the four trajectories shown in figure 6*b*, i.e. for the case where $C_{\mathrm{stab}}=C_{2x}=1200$ GtC.

Consider, first, the immediate-CPM trajectory. Along this trajectory, $\mathrm{d}C/\mathrm{d}t=(\mathrm{d}C/\mathrm{d}t)_0\cdot[1-(t-t_0)/\tau_{\mathrm{CPM}}]$. Using equation (2.3), the *ad hoc* Model, $\mathrm{d}C/\mathrm{d}t=\lambda\cdot[E(t)-E_{\mathrm{stab}}]$. Combining these two equations, the CPM trajectory for $E(t)$ is: $E(t)=[(\mathrm{d}C/\mathrm{d}t)_0/\lambda)\cdot[1-(t-t_0)/t_{\mathrm{CPM}}]+E_{\mathrm{stab}}$. It falls linearly from E_0 at time t_0 to E_{stab} at time $(t_0+\tau_{\mathrm{CPM}})$. Integrating this equation from $t=t_0$ to $t=t_0+\tau_{\mathrm{CPM}}$, we learn that the cumulative emissions during the stabilization period are $[E_{\mathrm{stab}}+E_0]\cdot\tau_{\mathrm{CPM}}/2$; in the same time period, the carbon content has climbed by $(\mathrm{d}C/\mathrm{d}t)_0\cdot\tau_{\mathrm{CPM}}/2$, which, by equation (3.4), is the entire headroom.

For $C_{\mathrm{stab}}=1200$ GtC, the CPM emissions trajectories produced by the two models are superimposed in figure 7. For the *ad hoc* Model, we have $\lambda=0.8$ and $E_{\mathrm{stab}}=3$ GtC yr^{-1} (see §3). The value of τ_{CPM} is still 200 years, since equation (3.4) is model-independent. So the CPM emissions trajectory falls linearly over 200 years from 8 to 3 GtC yr^{-1}. The cumulative emissions are 1100 GtC, and they result in an increase of 400 GtC in the atmosphere.

By contrast, the corresponding emissions trajectory for the Constant Airborne Fraction Model (shown in figure 6*b* and again in figure 7) falls from 8 GtC yr^{-1} all the way to zero after 200 years. This model predicts that only 800 GtC can be emitted into the atmosphere during the 200 years of its CPM stabilization trajectory. The extra 300 GtC of emissions allowed by the *ad hoc* Model, for the *same* 400 GtC headroom at the start, reflect the long-term sink present in the *ad hoc* Model but not in the Constant Airborne Fraction Model. Thanks to the long-term sink, a smaller fraction of emissions remains in the atmosphere. The long-term sink is real, and it is especially important when $E(t)$ is close to E_{stab} and falling. Accordingly, the *ad hoc* Model and its associated CPM trajectory are a better representation of CPM stabilization than the (easier to use) Constant Airborne Fraction Model and *its* CPM trajectory.

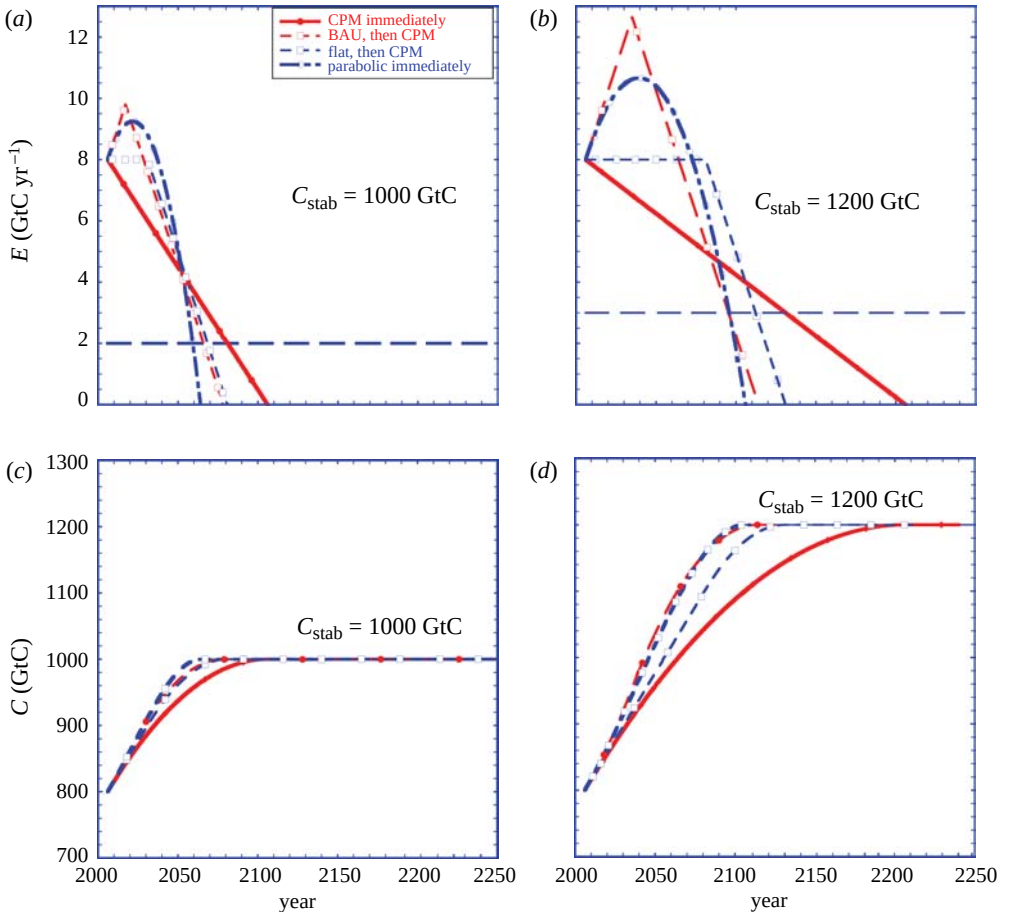

Figure 6. Four stabilization trajectories, $E(t)$, and their associated trajectories of atmospheric carbon content, $C(t)$. The $E(t)$ trajectories are computed using equation (2.1) and are shown for two values of the target stabilization level, $C_{\mathrm{stab}}=1000$ GtC (a,c) and $C_{\mathrm{stab}}=1200$ GtC (b,d). Because the Constant Airborne Fraction Model ($E_{\mathrm{stab}}=0$) is assumed, and $k=0.5$, the integrals under the four $E(t)$ trajectories in (a) are all 400 GtC, and under the trajectories in (b) they are all 800 GtC. In (a,b), a horizontal line is drawn at $E=E_{\mathrm{stab}}$, where E_{stab} is given by equation (2.2). The CPM case is discussed in §3 and the other three cases are discussed in §6*a–c*.

Second, consider the trajectory we are calling 'Business As Usual, then Constant Pace Mitigation' ('BAU, then CPM'). Here, we use *both* models. During the BAU period, we use the Constant Airborne Fraction Model, bowing to the evidence summarized in figure 2, obtained while the emissions rate has been rising. We believe that the Constant Airborne Fraction Model should continue to be reliable during any period when emissions are growing. However, at the peak of the emissions rate, when BAU gives way to CPM, we switch to the *ad hoc* Model. (We will explain this switch in §7; see especially figure 10.) The CPM trajectory descends to $E_{\mathrm{stab}}=3$ GtC yr^{-1}. The corresponding trajectories are very similar: the *ad hoc* trajectory peaks only 0.2 GtC yr^{-1} above the Constant Airborne Fraction trajectory (at 12.8 GtC yr^{-1} instead of 12.6 GtC yr^{-1}), and only one year later (after 30 years instead of 29 years). The *ad hoc* trajectory reaches the stabilization

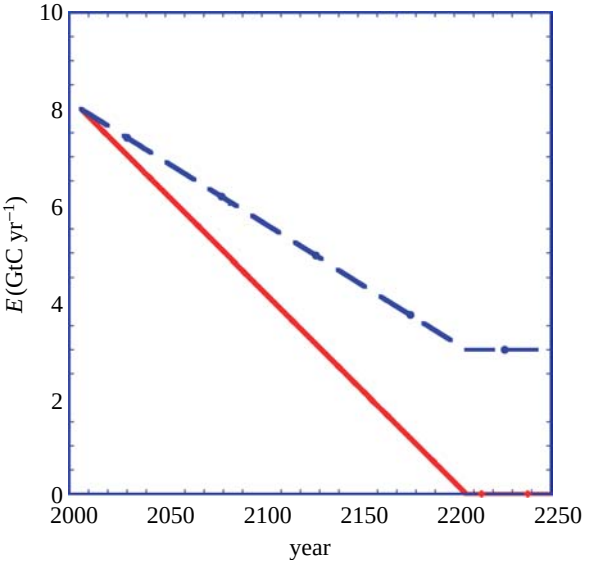

Figure 7. The CPM trajectories for $C_{\mathrm{stab}}=1200$ GtC, for the Constant Airborne Fraction Model (solid line) and the *ad hoc* Model (dashed line).

emissions level, 3 GtC yr^{-1}, after 92 years. The corresponding trajectory for the simpler Constant Airborne Fraction Model, shown in figure 6*b*, falls below 3 GtC yr^{-1} after 89 years and reaches zero after 108 years. Given the superiority of the *ad hoc* Model for falling emissions trajectories, we infer that the final two decades of the Constant Airborne Fraction trajectory should be discarded, but the first 90 years are 'good enough'.

Third, consider the emissions trajectory we are calling 'Flat, then CPM', with the magnitude of the pace during the CPM period constrained to be 0.16 GtC yr^{-1} per year. The trajectory motivated by the *ad hoc* Model remains longer in its flat phase and has a shorter CPM phase than the trajectory motivated by the Constant Airborne Fraction Model. Along both trajectories, the flat phase uses up 4 GtC of headroom each year. For the *ad hoc* Model, the reduction in the emissions rate is 5 GtC yr^{-1} during the CPM period, and therefore, the CPM period last only 31 years and consumes only the final 62 GtC of the 400 GtC of headroom. As a consequence, the flat phase can continue for 84 years. The corresponding trajectory for the Constant Airborne Fraction Model, as seen in §6.2, has a CPM period lasting 50 years and consuming the final 100 GtC of headroom, and therefore a flat phase lasting only 75 years. The *ad hoc* Model is more reliable in the CPM period.

Finally, for the case of parabolic mitigation, the parabolic emissions for the *ad hoc* Model are constrained to fall to 3 GtC yr^{-1} (rather than to zero), just as the headroom is fully used up. Since $P_{\mathrm{CPM}}=0.025$ GtC yr^{-1} per year, we find from equation (6.5) that $s=2.82$. The two corresponding trajectories are particularly similar for these inputs. The peak of the *ad hoc* emissions trajectory is about 0.1 GtC yr^{-1} lower and arrives less than one year earlier than the peak of the Constant Airborne Fraction Model. After 88 years, the *ad hoc* trajectory stops at 3 GtC yr^{-1}, while, as seen in figure 6*b*, the Constant Airborne Fraction trajectory continues for an additional decade, mostly below 3 GtC yr^{-1}.

The reader should bear in mind that both the Constant Airborne Fraction Model and the *ad hoc* Model are highly simplified. As explained in appendix A, some factors contributing to the uncertainty, such as the future use of land, are intrinsically unknowable. However, comparative statements can be made with considerable confidence: for accurate representations of changes in the atmospheric carbon content, if emissions trajectories are rising, use the Constant Airborne Fraction Model, and if they are falling, use the *ad hoc* Model.

(*e*) *The use of CPM parameters for year-to-year performance evaluation*

Global warming policy should evolve with time, as technology and other world conditions change. Thus, quantitative tools to assess year-by-year changes in parameters that measure global warming mitigation are important. Since policy assessment is most important during the years when $E(t)$ is rising, we adopt the Constant Airborne Fraction Model for the analysis. We return to the Constant Airborne Fraction Model. Taking the logarithmic derivatives of equation (3.6) and (4.2), we can manipulate the results to obtain the following:

$$\frac{1}{\tau_{CPM}}\frac{\mathrm{d}\tau_{\mathrm{CPM}}}{\mathrm{d}t} \simeq -\frac{1}{E}\frac{\mathrm{d}E}{\mathrm{d}t} - \frac{2}{\tau_{\mathrm{CPM}}} + \frac{1}{C_{\mathrm{stab}} - C}\frac{\mathrm{d}C_{\mathrm{stab}}}{\mathrm{d}t}, \tag{6.6}$$

$$\frac{1}{P_{CPM}}\frac{\mathrm{d}P_{\mathrm{CPM}}}{\mathrm{d}t} \simeq \frac{2}{E}\frac{\mathrm{d}E}{\mathrm{d}t} + \frac{2}{\tau_{\mathrm{CPM}}} - \frac{1}{C_{\mathrm{stab}} - C}\frac{\mathrm{d}C_{\mathrm{stab}}}{\mathrm{d}t}. \tag{6.7}$$

The derivatives on the right-hand side of these two equations can be evaluated using backward finite differences, using the data of the current year and the previous year. The derivatives on the left-hand side of these equations can be evaluated using forward finite differences. Thus the values of the two parameters $\tau_{\mathrm{CPM}}(t)$ and $P_{\mathrm{CPM}}(t)$ in the following year are obtained, and they inform us explicitly about the quantitative consequences of the performance of the mitigation strategy used in the previous year. The first equation tells us about the change in the time-constant parameter, τ_{CPM}, and the second equation tells us about the change in the pace parameter, P_{CPM}. If E_{stab} is not negligible in comparison to $E(t)$, then all the E's in equations (6.6) and (6.7) should be replaced by $E - E_{\mathrm{stab}}$. Note that equations (6.6) and (6.7) do not depend on k.

If the CPM strategy is followed, $P_{\mathrm{CPM}}(t)$ is constant. Finding that $P_{\mathrm{CPM}}(t)$ had increased relative to the previous year means we did less than what the CPM strategy demanded. An increase in next year's $P_{\mathrm{CPM}}(t)$ is the penalty. Similarly, when the CPM strategy is followed, $\tau_{\mathrm{CPM}}(t)$ decreases by one year per year—the deadline to finish the total emission reduction job for the currently chosen $C_{\mathrm{stab}}(t)$ does not change. If $\tau_{\mathrm{CPM}}(t)$ had decreased more quickly, the same deadline would have moved closer. For example, if $\mathrm{d}\tau_{\mathrm{CPM}}(t)/\mathrm{d}t = -5$, the deadline would have moved forward by four years.

Note that the factor $\mathrm{d}C_{\mathrm{stab}}/\mathrm{d}t$ in the third term on the right-hand side in both equations represents the revision of the stabilization target. Such a revision could result from new information (greater danger, new technology). Or, such a revision could result because the previous target had become unachievable, because of a constraint on the maximum pace of mitigation, as considered in §6*a*.

If P_{CPM} is fixed, so $\mathrm{d}P_{\mathrm{CPM}}/\mathrm{d}t=0$, equation (6.7) directly reveals by how much the value of C_{stab} must be revised. We obtain, after some algebra,

$$\frac{\mathrm{d}C_{\mathrm{stab}}}{\mathrm{d}t} \cong kE(t)\left(1+\frac{\mathrm{d}E/\mathrm{d}t}{P_{\mathrm{CPM}}}\right). \tag{6.8}$$

For example, with $P_{\mathrm{CPM}}=0.08$ GtC yr^{-1} per year at 2026, $(\mathrm{d}E/\mathrm{d}t)/P_{\mathrm{CPM}}$ is 2 and $E(t)$ is 11.2 GtC yr^{-1}. The slope of the dotted curve in figure 5 is therefore about 17 GtC yr^{-1}. In other words, the achievable C_{stab}, which figure 5 shows to be about 1300 GtC (620 p.p.m.), is rising by about 17 GtC yr^{-1} (8 p.p.m. per year).

In figure 8, we show the historical data for $[\tau_{CPM}(t)-\tau_{CPM}(t-1)]$, using equation (3.4) and the same data as we used to calculate $\tau_{\mathrm{CPM}}(t)$ in figure 3. The data would lie on a horizontal line at minus one, if the world had adopted a constant-pace-mitigation trajectory. Instead, we see that the deadline for achieving stabilization, for either $C_{\mathrm{stab}}=1200$ GtC or $C_{\mathrm{stab}}=1000$ GtC, has moved steadily closer in almost every year, except for two short periods: the deadline moved forward by more than a decade per year prior to 1970, and by about half a decade per year in recent years.

7. Modelling the long term sink and overshoot trajectories

In the Constant Airborne Fraction Model, based on correlation of empirical historical data, we assumed that a constant fraction of CO_2 emissions leaves the atmosphere quickly and the rest stays more or less forever. The *ad hoc* Model, equation (2.3), is an awkward attempt to achieve consistency with equation (2.2). Physically, there are *always* exchanges of CO_2 between the ocean and the atmosphere and between the terrestrial biosphere and the atmosphere, driving towards various physical, chemical and biological equilibria, even in the absence of fossil fuel CO_2 emissions. The ocean–atmosphere exchange is likely to be the dominant exchange on the multi-century time-scale, and it is expected to continue to remove CO_2 from the atmosphere. This removal process enables non-zero E_{stab} for a few centuries even after stabilization has been reached. The details of these processes were not needed in our analysis so far.

In the scientific literature, there are many 'stabilization' computer studies using science-based global carbon cycle models. One of the best known of these studies, by Wigley *et al.* (1996), produced the widely used WRE trajectories shown in figure 9. The emissions trajectories extend well into the period of stabilized concentration; for example, the 550 p.p.m. concentration trajectory reaches stabilization in about 2150. The corresponding 550 p.p.m. emissions from that date forward are the 'permitted' post-stabilization emissions that we are discussing here. We see that these permitted emissions after stabilization are about 4.5 GC yr^{-1} in 2150, fall to about 3.5 GtC yr^{-1} in 2200 and are less than 3 GtC yr^{-1} after 2300. The airborne fraction of these emissions is zero.

A finite, positive (but slowly decreasing in the century time-scale) E_{stab} is consistent with all science-based stabilization studies of the atmospheric concentration of CO_2. Finite E_{stab} means that CO_2 emissions from the fossil fuel system do not need to be completely eliminated after stabilization. Some endeavours can continue to burn fossil fuels and emit the CO_2 to the atmosphere. Ground and air transport and petrochemicals are likely contenders for these precious exemptions.

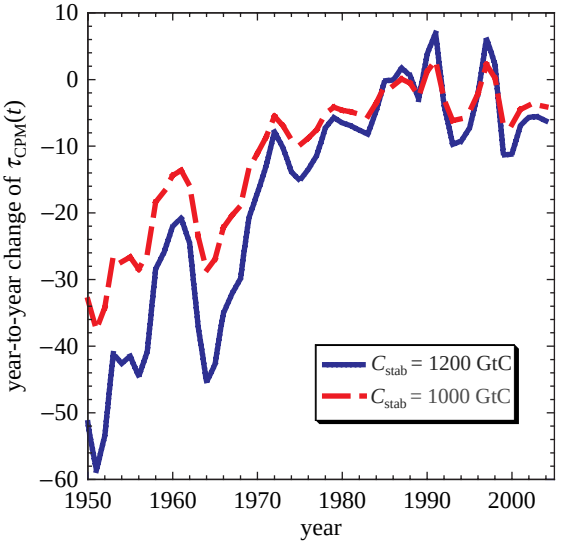

Figure 8. Historical data for $d\tau_{CPM}(t)/dt$, based on equation (3.5), for $C_{stab}=1200$ GtC (solid line) and $C_{stab}=1000$ GtC (dashed line).

(*a*) *The one-tank model*

To deal with scenarios in which E_{stab} is not negligibly small in comparison to $E(t)$, we introduce the *One-tank Model* to replace equation (2.1) and (2.3)

$$\frac{dC}{dt} = \kappa \cdot \left(E(t) - \frac{C - C_{pre}}{\tau_L} \right). \tag{7.1}$$

See appendix B for its derivation. Here, κ is a constant that plays the role of k and λ, τ_L is a *long* time constant (in units of years) measured in centuries and C_{pre} is the pre-industrial carbon content of the atmosphere, 600 GtC. Note that the exact analytical solution of equation (7.1)—a linear constant coefficient first order ordinary differential equation—for arbitrary $E(t)$ is well known. The One-tank Model adds a new term to the Constant Airborne Fraction Model, equation (2.1). It can also be considered a generalization of the *ad hoc* Model, equation (2.3), replacing E_{stab} by a term that depends on $C(t)$ and τ_L.

With the same pair of parameters, κ and τ_L (and C_{pre} fixed at 600 GtC), the One-tank Model can approximately emulate both the short- and long-term behaviour of many simulations of science-based models reported by the Intergovernmental Panel on Climate Change (2001). The new term on the right hand side of equation (7.1) can be interpreted as modelling a long-term sink, representing the deep ocean exchanging CO_2 with the rest of the ocean-atmosphere system with a characteristic exchange time, τ_L. The expression in the numerator of this term, $C(t) - C_{pre}$, is the difference between the atmosphere's carbon content at time t and in pre-industrial times. This factor can be understood to model the overturning of the ocean, which brings deep water to the surface that was last at the surface at some pre-industrial time, when it came into equilibrium with the atmosphere of that time. This water newly arriving at the surface has retained a memory of the earlier atmosphere, so that, when it returns to the surface, the efficacy of the ocean's removal of CO_2 from the atmosphere is

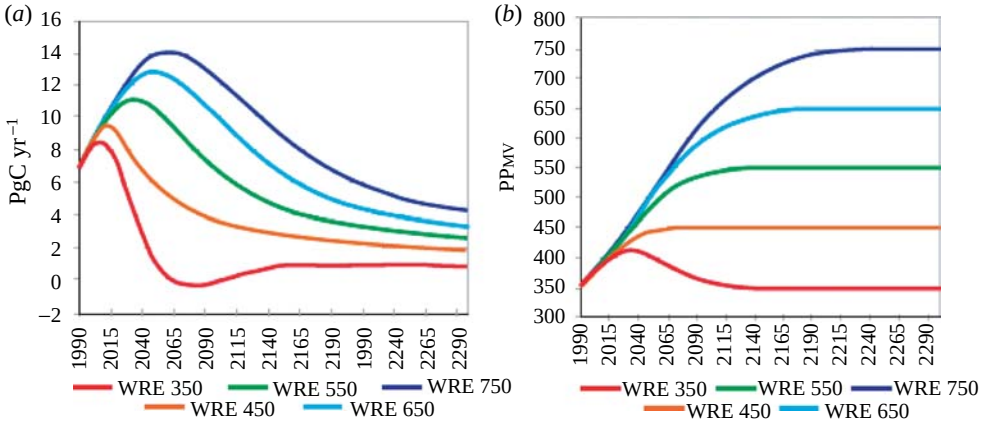

Figure 9. One version of the widely used WRE stabilization trajectories (Wigley *et al.* 1996). Illustrative CO_2 emissions profiles and corresponding concentrations. (*a*) Emissions scenarios from fossil fuel use and industrial activities; (*b*) corresponding atmospheric concentration levels. In this paper, we consider mostly the right end of the two panels, where stabilized concentrations are compatible with substantial finite emissions (US Department of Energy 2006, p. 36).

proportional to the difference between the current carbon content of the atmosphere and its content at the earlier time. In a longer time-scale, we would expect to need to replace the quantity C_{pre} with a function that rises slowly with time. Appendix B develops these ideas mathematically.

After stabilization, the left-hand side of equation (7.1) vanishes, and the stabilized emissions rate, $E_{\mathrm{stab}}(t)$, can be readily solved for

$$E_{\mathrm{stab}}(t) = \frac{C_{\mathrm{stab}} - C_{\mathrm{pre}}}{\tau_{\mathrm{L}}(t)}. \tag{7.2}$$

Thus, equation (2.2) is simply equation (7.2) with $\tau_{\mathrm{L}} = 200$ years. In equation (7.2), we allow for the possibility that τ_{L} may depend on time. This option is further exploited in appendix B.

We can extract information on $\tau_{\mathrm{L}}(t)$ from stabilization data generated by any credible carbon cycle model. Examining the region where C is already stabilized, we find in figure 9 that $E_{\mathrm{stab}}(t)$ is a slowly decreasing function of time for each specified C_{stab}. The empirical $\tau_{\mathrm{L}}(t)$ extracted from $E_{\mathrm{stab}}(t)$ using equation (7.2) is found to be approximately *independent* of C_{stab} and is a slowly increasing function of time (by a fraction of a year per year). For the sake of simplicity, we recommend in equation (2.2), the use of its average value of the next few centuries:

$$\tau_{\mathrm{L}}(t) \approx 200 \text{ years}. \tag{7.3}$$

Empirically, the value of $\tau_{\mathrm{L}}(t)$ is smaller before 2300 and larger after 2300. Using this constant value in equation (7.2) gives E_{stab} close to 2 and 3 GtC yr^{-1}, for $C_{\mathrm{stab}} = 1000$ and 1200 GtC, respectively.

(*b*) *Relations between the three models*

We are reminded that the Constant Airborne Fraction Model and the *ad hoc* Model are empirical correlations of available data. Only the One-tank Model is a model in the classical sense of models. Indeed, we can use the One-tank Model to

compute the solution $C(t)$ for any specified $E(t)$. Figure 10 shows two plots of $\mathrm{d}C/\mathrm{d}t$ versus E generated by solutions of the One-tank Model for two $E(t)$ trajectories, both specified by equation (6.1). C_{stab} is 1200 GtC for both cases. The 'CPM immediately' $E(t)$ trajectory has $s=-1$ (it is the dashed line trajectory in figure 7), and the 'parabolic immediately' $E(t)$ trajectory has some positive s so that $E(t)$ rises before falling toward E_{stab}. With $\tau=0.5$ and $\tau_{\mathrm{L}}=200$ years, both cases share the same starting point ($E=8$, $\mathrm{d}C/\mathrm{d}t=3.5$) and the same end point ($E=3$, $\mathrm{d}C/\mathrm{d}t=0$). Note that on this graph the Constant Airborne Model is a straight line passing through the origin, and the *ad hoc* Model is a straight line passing through the end point. The slightly time-dependent values of k (when E is rising) and λ (when E is falling) can be discerned by inspection of the slight curvatures of these two paths.

When a CPM $E(t)$ trajectory is actually implemented, the response of $C(t)$ of the 'real' global carbon cycle will not, in general, be strictly linear with respect to time. In fact, the exact response of $C(t)$ according to the One-tank Model will include a small nonlinear time-dependence which can be mathematically derived. Thus the headroom is only approximately the area of the triangle we talked about in §2. Consequently, the factor 2 in equations (3.3), (3.4) and (3.6) is an approximation. We recommend replacing this factor by the symbol f_{CPM}, which is nominally 2. As such, the resulting $C(t)$ trajectory will slightly undershoot the C_{stab} target. See the end of appendix B for further comments on this point.

We can show that the Constant Airborne Fraction Model is the model of choice when $E(t)$ is moving away from E_{stab}, and that the *ad hoc* Model is the model of choice when $E(t)$ is moving towards E_{stab}. The strength of the Constant Airborne Fraction Model is its ease of use during the procrastination period. The strength of the *ad hoc* Model is its consistency near the end of the mitigation period. The strength of the One-tank Model is its applicability in all time domains of interest—at the price of being somewhat less easy to use. Near the end of appendix B, we shall provide some support for these claims. Of particular interest is that $E_{\mathrm{stab}}(t)$ after stabilization is expected to decrease slowly with time, and this feature can be accounted for by allowing $\tau_{\mathrm{L}}(t)$ to be time dependent in the One-tank Model.

(*c*) *Overshoots*

If we approach C_{stab} from below, $E(t)$ approaches E_{stab} from above. However, if $C(t)$ has already breached the desired C_{stab} level, some future $E(t)$ must be less than E_{stab} in order to push $C(t)$ back down to C_{stab}. In other words, in order to recover after C_{stab} is breached, the right-hand side of equation (7.1) must be negative, which requires the newly added second term in the one-tank model.

Trajectories that peak and then fall back to a target concentration are called 'overshoot' trajectories (O'Neill & Oppenheimer 2004; Stern 2007; Wigley *et al.* 2007). An example of an overshoot trajectory is the 350 p.p.m. concentration trajectory in figure 9, which peaks at 420 p.p.m. in 2030 before falling to 350 p.p.m. about a century later. In that entire century, emissions are below the $E_{\mathrm{stab}}(t)$ for 350 p.p.m. (Emissions are actually *negative* in the last third of this century.)

Let us work an example to estimate how low emissions must be in order to recover from an overshoot trajectory. Suppose $C(t)$ has peaked at the plateau of the 550 p.p.m. stabilization trajectory in 2150, following the path shown in

figure 9, and then the world decides that to embark on a '450 p.p.m. stabilization programme' from above, with the goal of descending to 450 p.p.m. over some period of time. Using the Rosetta stone, the carbon content of the atmosphere needs to drop from about 1200 GtC to about 1000 GtC, i.e. 200 GtC must be removed from the atmosphere.

One strategy would be simply to lower $E(t)$ quickly onto the stabilization trajectory for 450 p.p.m. and to remain on this trajectory while the atmospheric carbon content falls. The exact analytical solution of equation (7.1) says the concentration will fall approximately exponentially from 550 to 450 p.p.m., with an *e*-folding time of $\tau_{\mathrm{L}}/k \approx 400$ years; after 800 years, about 85 % of the gap will have closed. In short, even with a quick drop from $E = 4.5\ \mathrm{GtC\ yr^{-1}}$ to the new E_{stab} (which is about $3\ \mathrm{GtC\ yr^{-1}}$ in 2150), the recovery is very, very slow. The important point is that, as long as $E(t)$ is positive, the speed of the recovery is not controlled by the speed of the drop of $E(t)$; it is controlled by the long-term sink in the carbon cycle model.

Instead of equation (7.1), we can use figure 9. In 2150, as just noted, stabilization emissions on the 550 p.p.m. trajectory are about $4.5\ \mathrm{GtC\ yr^{-1}}$ and about $3.0\ \mathrm{GtC\ yr^{-1}}$ on the 450 p.p.m. trajectory. These numbers provide the magnitude of the long-term sink. If the world is in a greater hurry, emissions will have to be brought *below* the value of $E_{\mathrm{stab}}(t)$ for 450 p.p.m. so that the long term sink can overpower the emissions. Since only half (for $\kappa = 0.5$) of the reduced emissions will affect the atmosphere, the needed emissions trajectory over some period of time must result in integrated emissions roughly 400 GtC less than those of $E_{\mathrm{stab}}(t)$ for 450 p.p.m.

Consulting figure 9, to compensate for the overshoot, the emissions path could descend rapidly from the 450 p.p.m. E_{stab} curve to a value $2\ \mathrm{GtC\ yr^{-1}}$ below this curve, remain $2\ \mathrm{GtC\ yr^{-1}}$ below for 200 years, and then return rapidly to this curve, by then considerably lower. Alternatively, the path could spend a shorter time further below the 450 p.p.m. E_{stab} curve—100 years a full $4\ \mathrm{GtC\ yr^{-1}}$ below, for example. Since the emissions curve in figure 9 for 450 p.p.m. is only $3\ \mathrm{GtC\ yr^{-1}}$ in 2150 and $2\ \mathrm{GtC\ yr^{-1}}$ in 2300, we see that emissions will be near zero for most of the time when the world spreads the job over two centuries, and the emissions need to be negative for some of the time when the world rushes to get the job done in just one century.

Proposals for overshoot trajectories tend to underestimate the difficulty of lowering the emissions rate below E_{stab}, which is already quite low for all interesting C_{stab}. In general, the marginal cost of pollutant removal in a pollution control programme increases steeply as the reduction approaches 100%. In the case of mitigation of CO_2 emissions, a steep mitigation pace may be viable in the early stages when opportunities for emissions reduction are plentiful. However, at the late stages of mitigation, wringing the last few $\mathrm{GtC\ yr^{-1}}$ of fossil fuel emissions out of the world economy can be expected to be very difficult. The transportation sector, in particular, can be expected to be very grateful for the finite positive E_{stab} permitted after stabilization.[4]

[4] The recently issued Stern report warns that overshoot trajectories 'would be both practically very difficult and very unwise. Overshooting paths involve greater risks, as temperatures will also rise rapidly and peak at a higher level for many decades before falling back down. Also, overshooting requires that emissions subsequently be reduced to extremely low levels, below the level of natural carbon absorption, which may not be feasible' (Stern 2007, p. xi).

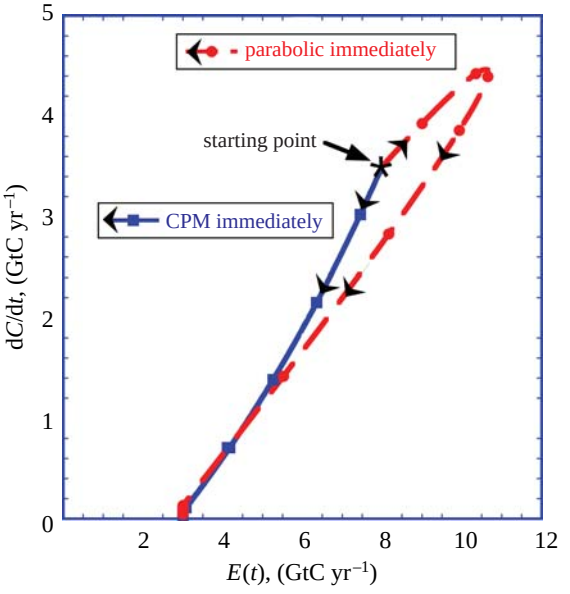

Figure 10. Two $\mathrm{d}C/\mathrm{d}t$ versus E plots using solutions generated by the One-tank Model with $\tau = 0.5$ and $\tau_{\mathrm{L}} = 200$ years. C_{stab} is 1200 GtC and E_{stab} is $3\,\mathrm{GtC\,yr^{-1}}$. The solid line is the 'CPM immediately' case and the dashed line is the 'parabolic immediately' case (see text). The asterisk shows the common starting point for the two paths.

Relief from the tyranny of rapidly rising costs for near-zero emissions may become available, if significant quantities of CO_2 can be removed from the atmosphere. CO_2 removal is accomplished, for example, when biomass is grown sustainably and used as a power plant fuel, with the power plant's CO_2 emissions captured and stored in geological formations. Someday, perhaps, CO_2 may be captured directly from the air in significant quantities by chemical processes (Elliott *et al.* 2001; Zeman and Lackner 2004; Keith *et al.* 2005; Stolaroff *et al.* 2006; Zeman 2006). Indeed, even net negative global CO_2 emissions are conceivable by such strategies, but the task is formidable, even over the very long term.

8. Are these tools good enough for policy making?

The tools provided so far depend only on two empirical parameters. Equation (3.5) for $\tau_{\mathrm{CPM}}(t)$ needs a multiplicative time-scaling parameter for $\mathrm{d}C/\mathrm{d}t$ (either k for equation (2.1), or λ for equation (2.3), or κ for equation (7.1)), and equation (2.2) for E_{stab} needs τ_{L}. Our tools load all interactions with the biosphere and shallow ocean onto the time-scaling parameter; many of these interactions are poorly understood, and some can be modified by future human decisions, especially decisions about land use. The value of τ_{L} depends mostly on the slow dynamics of the ocean, which no human activity can significantly modify.

In §1 we specified the emissions to be included in $E(t)$, and in §§2 and 7, respectively, we made recommendations for the values of the time-scaling parameter and for τ_{L}. Modellers are welcome to make other choices about what to include in $E(t)$ and to use other numbers. When someday an improved carbon cycle model appears,

the crucial questions from the policy point of view will be: what is the new time-scaling parameter and what is τ_{L}? The policy consequences of the improvements in the new model can easily be assessed using the tools provided here.

We used the Constant Airborne Fraction Model for first-approximation back-of-the-envelop policy calculations. In particular, we made use of equation (4.2), a simplified version of equation (4.1) because it neglects E_{stab}.

(*a*) *Informing the public*

We wish to promote the use of $\tau_{\mathrm{CPM}}(t)$ as defined by equation (3.5), which contains no modelling parameters, as a useful measure of the state of global warming. We recommend that the world develop some reporting mechanisms, perhaps through the United Nations, to publicize the value of $\tau_{\mathrm{CPM}}(t)$ annually, so all can keep track of its value and its annual changes. Prior to the existence of an agreed specific C_{stab} target level, the $\tau_{\mathrm{CPM}}(t)$'s for a small number of representative target levels could be reported annually.

We also recommend that the public report include the database of the historical $\tau_{\mathrm{CPM}}(t)$ trajectory of the past several decades. The data could be presented in graphs which resemble continually updated versions of figures 3 and 8. If emissions indeed follow the CPM strategy, the date when the whole job is done would stay the same. Thus when τ_{CPM} shortens by one year each year, we are on course for the CPM strategy. If $\tau_{\mathrm{CPM}}(t)$ shortens by more than one year per year, the world is not working to reduce emissions as hard as is required by the CPM strategy. If $\tau_{\mathrm{CPM}}(t)$ decreases by less than one year each year, or even increases, good progress is being made, and the good report may prompt us to consider a lower value of C_{stab}.

9. Conclusions

We provide a simple framework for discussing of the objective of stabilization of the CO_2 concentration (equivalently, carbon content) in the atmosphere. We find that for a stabilization target at double the pre-industrial level, C_{2x}, the total fractional reduction of the rate of fossil-fuel CO_2 emissions relative to its current rate is more than two-thirds. We also find that, if we begin immediately, we can spread out the stabilization task over approximately two centuries. After stabilization, a small but finite emissions rate for fossil fuel CO_2 is allowed for several centuries.

We propose a benchmark strategy of mitigation, the 'CPM' strategy, where the annual emissions rate is reduced by a constant amount each year, until it falls to E_{stab} at just the moment when the concentration reaches the target stabilization concentration (i.e. the 'headroom' is used up). We recommend a simple empirical formula for E_{stab}, good enough for the next few centuries. We introduce the concept of $\tau_{\mathrm{CPM}}(t)$, the time-scale of the CPM strategy, and define it in such a way that it is independent of any modelling parameters or assumptions. The value of $\tau_{\mathrm{CPM}}(t)$ can be computed at any time using only observational data for $C(t)$; its dependence on the target C_{stab} is explicitly provided. Only two modelling parameters are involved, a multiplicative time-scaling parameter [linking $E(t)$ to $\mathrm{d}C/\mathrm{d}t$] and a time, τ_{L}, used to estimate the value of E_{stab}. If the goal is to keep the carbon content of the atmosphere below C_{2x}, and the CPM strategy begins immediately, we find with our recommended values that $\tau_{\mathrm{CPM}}(t)$ could extend over roughly 200 years. Modellers who prefer other values for these two parameters are encouraged to use them.

We introduce the concept of a maximum pace of mitigation, $P_{\mathrm{CPM}}^{\max}$, as the maximum year-to-year reduction in the rate of fossil fuel CO_2 emissions judged to be achievable. We quantify an n-wedge effort by adapting the measure introduced by Pacala and Socolow, where a single-wedge effort changes the direction of an emissions trajectory by $P_{\mathrm{CPM}} = 0.02\ \mathrm{GtC\ yr^{-1}}$ per year (Pacala & Socolow 2004). We present some calculations for two-wedge, four-wedge, and eight-wedge efforts, showing how the floor on achievable values of C_{stab} depends on $P_{\mathrm{CPM}}^{\max}$ and the starting value of $E(t)$. We observe that if the world right now were to begin a CPM strategy intended to prevent the carbon content of the atmosphere from exceeding C_{2x}, a two-wedge effort would suffice, but one that would need to be conscientiously maintained for the next two centuries.

Procrastination, defined to be delaying the start of the CPM strategy, can quickly turn mitigation from a task of centuries to a task of only a few decades. We provide a simple way to assess year-to-year progress in dealing with global warming or year-to-year consequences of ignoring the problem. If we procrastinate for the next 50 years with BAU, the emission rate would have doubled and the option of preventing the carbon content of the atmosphere from reaching C_{2x} would be foreclosed, in the sense that the necessary P_{CPM} over the few CPM decades after the procrastination would be unrealistically large.

We recommend that a credible organization (such as the United Nations) should publicize annually the values of $\tau_{\mathrm{CPM}}(t)$ for several interesting stabilization targets, and make the underlying databases accessible to the general public via the internet. We believe the information conveyed by these data can greatly enhance the quality of public discourse on this subject.

The authors would like to thank Bryan Mignone for pointing the way on procrastination issues, Bryan Mignone and Jeffery Greenblatt for their patience in introducing us to ocean modelling, Thomas Kreutz for calling our attention to Nordhaus's box models of the ocean, and Tom Wigley for helpful discussions of figure 9. We have also benefited from discussions with Anand Gnanadesikan, Stephen Pacala, and Jorge Sarmiento, close readings of manuscript drafts by Shoibal Chakravarty and Xu Yuan, and encouragement from Barrie Royce and Fred Dryer.

Appendix A. Supporting material about the Earth's environment

Details of the calculation for the Rosetta stone: 2.1 GtC = 1 p.p.m. The atmospheric CO_2 content, when expressed in units of GtC, represents the total mass of carbon in the atmosphere, while its atmospheric concentration, when expressed in units of p.p.m., is the fraction of the molecules in the atmosphere that are CO_2 molecules. The Rosetta stone connects these two numbers.

Finding the Rosetta stone requires only three inputs.

(i) Total dry mass of atmosphere[5]: 5.1×10^{18} kg.
(ii) Average molecular weight of air in the atmosphere: 29.
(iii) Atomic weight of carbon: 12.

[5] It is conventional to report concentrations excluding water vapour, because the water vapour content of the atmosphere is highly variable. On average, there are about 1.3×10^{16} kg, or about 7×10^{14} kg-moles, of water vapour in the atmosphere, or four parts per thousand by volume.

Dividing input (i) by input (ii), the atmosphere (excluding water vapour) contains 1.8×10^{17} kg-moles of gas, and one part per million (by volume) contains 1.8×10^{11} kg-moles of gas.[6] Hence, multiplying by input (iii), each p.p.m. of molecules containing carbon has a carbon mass of 2.1×10^{12} kg, or 2.1 billion metric tons (Gt):

$$1 \text{ p.p.m.} = 2.1 \text{ GtC},$$

which is quoted near the end of §1.

The 'Divide by Four' rule Combining the Rosetta stone and the Constant Airborne Fraction Model, with $k=0.5$, results in another useful rule of thumb: the *'Divide by Four'* rule. The carbon burned, in GtC units, is four times the atmospheric CO_2 increase, in p.p.m. units. If 4 GtC of fossil fuels are removed from below ground and burned, the Constant Airborne Fraction Model with $k=0.5$ says only about 2 GtC stays in the atmosphere, and the Rosetta stone tells us that the atmospheric concentration will climb by about 1 p.p.m. Using this rule of thumb, burning of 8 billions tons of carbon per year, as we do today, produces a 2 p.p.m. rise per year, which is approximately the current rate of climb in the atmospheric concentration of CO_2.

The carbon balance of the atmosphere since 1751 The total of all fossil fuel emissions from 1751 to 2004 is 305 GtC (Marland *et al.* 2006). We would have guessed that 400 GtC had been added, using the Constant Airborne Fraction Model with $k=0.5$ and knowing that the atmospheric carbon content has grown by 200 GtC. Looking more closely, about half of the 305 GtC of emissions occurred after 1980 (when the concentration was 339 p.p.m. and the carbon content was about 710 GtC) and three quarters after 1960 (when the concentration was 317 p.p.m. and the carbon content was 670 GtC). These data confirm what we already know from figure 1: the Constant Airborne Fraction Model with $k=0.5$ closely approximates (in fact, slightly underestimates) fossil fuel CO_2 retention after 1960. But for the period between 1800 and 1960, these data carry a surprise: the rise in the atmospheric CO_2 concentration, roughly 70 GtC, is nearly equal to the total fossil fuel emissions (one quarter of 305 GtC), i.e. the airborne fraction was almost one (Marland *et al.* 2006). Prior to 1940, the land biosphere acted as a source of CO_2, nearly cancelling the ocean sink (Sarmiento *et al.* 1992).

The emissions rate since 1900 The fossil fuel CO_2 emission rate climbed relatively slowly before 1960. It was within 20% of 1 GtC yr^{-1} for thirty years between 1910 and 1939, and was within 20% of 2 GtC yr^{-1} between 1950 and 1958. Emission rates grew twice as fast between 1960 and 1980 as between 1980 and 2000, 0.14 versus 0.07 GtC yr^{-2}. The emissions rate actually fell four times: from 1974 to 1975, from 1979 to 1983, from 1991 to 1992, and from 1997 to 1999. The one-year rise between 2002 and 2003, 0.33 GtC yr^{-2} (from 6.97 to 7.30 GtC yr^{-1}) was larger than any previous one-year rise (Marland *et al.* 2006).

The approximate strengths of the ocean and land sinks recently The oceans continually exchange CO_2 with the atmosphere. Terrestrial plants also exchange CO_2 with the atmosphere as plants grow, decay or burn, and as forests are deliberately cut down and others are deliberately established. In round numbers, 8 GtC yr^{-1} are being added to the atmosphere by burning fossil fuels today and the atmosphere's carbon content is increasing by 4 GtC yr^{-1}. In recent decades a

[6] One kg-mole is 1000 times Avogadro's number of molecules, i.e. 6×10^{26} molecules.

net amount of 2 GtC yr^{-1} was removed by the oceans, and this rate is slowly increasing. A net amount of probably 1 to 2 GtC yr^{-1} is being removed by the terrestrial biosphere, in spite of a deforestation rate (a *source* of carbon for the atmosphere) today probably bracketed by 1 and 2 GtC yr^{-1}. Evidently, terrestrial biological mechanisms that remove carbon from the atmosphere are, in aggregate, roughly twice as large in magnitude as deforestation. Candidates include results of past land use, for example regrowth of forests previously cut down, and enhanced growth of plants in a higher-CO_2 atmosphere.

Year-to-year variability in fraction of emissions retained by the atmosphere It is clear from figure 1 that the fraction of fossil fuel retained in the atmosphere varies greatly from year to year. It is now known that most of this variability comes from year-to-year changes in the exchange between the terrestrial biosphere and the atmosphere; the strength of the ocean sink changes only slowly from year to year.

Experts are quite confident about the future strength of the ocean sink, thanks to a combination of measurements of tracers (like carbon-14) throughout the oceans and sophisticated General Circulation Models (Matsumoto *et al.* 2004). But the net exchange of CO_2 between the biosphere and the atmosphere (the net 'land sink') is an entirely different matter. Uncertainty about the future land sink arises from both limited understanding of plant physiology in a higher and warmer CO_2 world, and from land-use decisions by human beings that are still to be decided, for example, future decisions about deforestation and replanting. Even the sign of the future net exchange between the terrestrial biosphere and the atmosphere is uncertain. As a result, not surprisingly, there is considerable disagreement among experts on the future strength of the land sink. Minimalist models using aggregate sinks, such as those in this paper, are justified above all, in our opinion, by this uncertainty about the land sink. Models that combine a precise ocean sink with a crude land sink can obscure many interesting 'good enough' results.

Average surface-temperature increase Today, targets are frequently expressed in terms of a maximum rise in the average surface temperature, relative to pre-industrial times, rather than as CO_2 concentrations. The 1000 and 1200 GtC carbon targets are understood in political discourse to be roughly equivalent to 2 and 3°C temperature targets, respectively. However, political discourse is taking a short-cut. Climate models cannot yet establish a direct correspondence between temperature rise and concentration increase. Instead, there are probability statements. For example, stabilization at twice the pre-industrial concentration is predicted to have approximately a 50% chance of preventing a long-term rise of 3°C in average surface temperature, and only a 15% chance of preventing a long-term rise of 2°C (Meinshausen 2006).

The preference in current political discourse for expressing stabilization goals in terms of maximum allowed temperature rise, rather than maximum stabilization concentration, may be regretted some day. Driving those who have chosen to express stabilization goals in terms of temperature rise, apparently, is the belief that the citizen can relate to warming but not to concentration build-up. Traded against this benefit, however, is a loss of clarity. Owing to the probabilistic relationship between concentration and average surface temperature, one is condemning diplomats to argue about probabilities, not obviously a promising prospect. There is obvious merit in returning to targets expressed in p.p.m. or GtC.

Aerosols and other greenhouse gases Completeness requires our notifying the reader that CO_2 is the major greenhouse gas which contributes to global warming, but not the only one. Methane (CH_4) and nitrous oxide (N_2O) are the two next most important; their emissions are dominated by anaerobic processes on the Earth's surface, in wetlands and on irrigated lands, in the stomachs of cows and on the fertilized field. Aerosols are also high on the list of substances that will need to be managed alongside CO_2; their interactions with climate are particularly complex. For further information, see Intergovernmental Panel on Climate Change (2001).

Appendix B. Modelling global carbon cycles

The response of the global average atmospheric carbon content, C, to the trajectory of global fossil-carbon emission rate, E, is the responsibility of global carbon cycle models. Such models depend on detailed descriptions of the physics and chemistry involved, and are usually highly complex and nonlinear. Nevertheless, the linear concept of the Green's function is often used to represent the response of computer simulations. We shall show how to extract from any given Green's function the equivalent mathematical model, including the values of the needed parameters. This process is sometimes called reverse-engineering.

(*a*) *The Green's function*

When a unit pulse of carbon is released to the atmosphere at $t=0$, the time evolution of the subsequent atmospheric carbon content is called the Green's function of the system. We shall analyse one such Green's function, developed to describe the ocean–atmosphere system with a carbon-neutral biosphere (Sarmiento *et al.* 1992; see also Joos *et al.* 1996). The Sarmiento Green's function has five modes

$$G(t) = \sum_{n=0}^{4} A_n \exp\left(-\frac{t}{\tau_n}\right), \tag{B 1}$$

where t is time (year), and

$$\begin{aligned} A_0 &= 0.174, \quad t_0 = \infty, \\ A_1 &= 0.275, \quad t_1 = 376.6 \text{ years}, \\ A_2 &= 0.307, \quad t_2 = 67.7 \text{ years}, \\ A_3 &= 0.189, \quad t_3 = 10.7 \text{ years}, \\ A_4 &= 0.0545, \quad t_4 = 0.9 \text{ years}. \end{aligned}$$

Note that $G(t=0^+)=1$.

If we consider only the time interval $t \gg t_2$ and ignore the 'fast transients', then the Sarmiento Green's function simplifies to

$$G(t) \cong A_+ + A_1 \exp\left(-\frac{t}{\tau_1}\right). \tag{B 2}$$

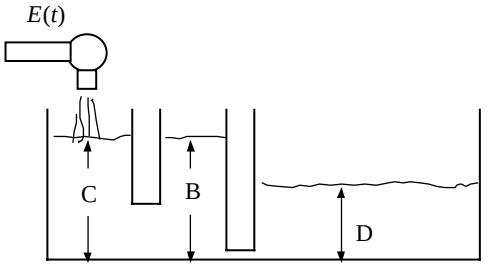

Figure 11. The three-tank model of global carbon cycle.

(*b*) *The three-tank model*

Consider the following system of three linear, constant-coefficient first-order ordinary differential equations (Nordhaus 1994; Nordhaus & Boyer 1999, eqns 3.8, 3.9, and 3.10):

$$\frac{\mathrm{d}C}{\mathrm{d}t} = -\frac{C-B}{\tau_{\mathrm{S}}} + E, \tag{B 3}$$

$$\beta\frac{\mathrm{d}B}{\mathrm{d}t} = +\frac{C-B}{\tau_{\mathrm{S}}} - \frac{B-D}{\tau_{\mathrm{L}}}, \tag{B 4}$$

$$\gamma\frac{\mathrm{d}D}{\mathrm{d}t} = +\frac{B-D}{\tau_{\mathrm{L}}}. \tag{B 5}$$

Here E represents the annual amount of carbon entering the atmosphere from reservoirs, which are normally totally isolated from the atmosphere (such as fossil fuels below the ground), and C, B, and D are the carbon contents of the atmosphere, the biosphere/shallow ocean, and the deep ocean, respectively. This *three-tank model* has four parameters: τ_{S}, τ_{L}, β and γ. All are assumed to be positive constants. The efficacy of carbon exchange between the C and B tanks is characterized by τ_{S}, a *short-term* time scale, while the similar time-scale between the B and D tanks is characterized by τ_{L}, which is a *long-term* time-scale. Figure 11 graphically represents the disparity of the two time scales by the size of the tank connectors. The capacity of each tank to store carbon, normalized to that of the C tank, is represented by β and γ, which are dimensionless. Large β or γ means the associated tank has massive capacity to store carbon.

Adding equation (B 3) and equation (B 4), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(C+\beta B) = E - \frac{B-D}{\tau_{\mathrm{L}}}, \tag{B 6}$$

which shall be found useful immediately below.

(*c*) *The two-tank model*

When the $E(t)$ trajectory is smooth, we intuitively expect C and B to nearly equilibrate, while the drain-off to the D tank proceeds very slowly. Exploiting this observation, we approximate B by C and rewrite equation (B 6) and equation (B 5) as follows:

$$\frac{\mathrm{d}C}{\mathrm{d}t} \cong \kappa\left(E - \frac{C-D)}{\tau_{\mathrm{L}}}\right), \tag{B 7}$$

$$\frac{\mathrm{d}D}{\mathrm{d}t} \cong +\frac{1}{\gamma}\,\frac{C-D}{\tau_{\mathrm{L}}}, \tag{B 8}$$

where $\kappa = 1/(1+\beta)$. If the deep ocean is assumed to have a massive capacity to store carbon, then $\gamma \gg 1$, and we conclude from equation (B 8) that D is nearly a constant. Thus equation (B 7) recovers equation (7.1) of the main text. We shall call equation (B 7) and (B 8) the *two-tank* model.

(*d*) *Values of parameters from the Sarmiento Green's function*

The Sarmiento model is, of course, much more detailed and sophisticated than the three-tank and two-tank models. The simple models do not provide values for the needed parameters, while the Sarmiento model provides all the needed numbers.

For the sake of simplicity, we shall concentrate on the two-tank model from here on. The two eigenvalues for the two-tank model are zero and $-(\kappa+1/\gamma)/\tau_{\mathrm{L}}$. The Green's function $G(t)$ for this finite-tank system is easily obtained using elementary mathematics with $E(t)$ replaced by a unit pulse at $t=0$

$$G(t) = \frac{\kappa}{1+\kappa\gamma} + \frac{\kappa^2\gamma}{1+\kappa\gamma}\exp\left(-\frac{1+\kappa\gamma}{\gamma}\,\frac{t}{\tau_{\mathrm{L}}}\right).$$

Comparing it to the simplified version of the Sarmiento Green's function, equation (B 2), we have

$$\frac{\kappa}{1+\kappa\gamma} = 0.174, \quad \frac{\kappa^2\gamma}{1+\kappa\gamma} = 0.275, \quad \frac{\gamma\tau_{\mathrm{L}}}{1+\kappa\gamma} = 376.6 \ \text{ years}.$$

Solving these three equations for the three constants, we obtain

$$\kappa = 0.45, \quad \tau_{\mathrm{L}} = 275 \ \text{ years}, \quad \gamma = 3.6.$$

The associated value for β is $(1-\kappa)/\kappa = 1.2$. In other words, the two-tank model with these numbers is an emulator of the Sarmiento model for $t \gg t_2 \sim 67.7$ years—after the rapid transients have died. The value of τ_{S} is not needed, but the Sarmiento Green's function indicates that it is roughly 70 years. So the two-tank model is formally applicable only for $t \gg 70$ years.

Note that γ—as determined from the Sarmiento Green's function—is only a moderately large number. Certainly the reciprocal of 3.6 is not a small enough number. Thus in the two-tank model, we expect D to be a slowly rising function of time.

(*e*) *The one-tank model*

When t/τ_1 is small, the exponential term in the simplified Sarmiento Green's function, equation (B 2), can be expanded in a Taylor series. We have

$$\begin{aligned} G(t) &= 0.174 + 0.275\exp\left(-\frac{t}{\tau_1}\right) \cong 0.174 + 0.275\left(1-\frac{t}{\tau_1}+\cdots\right) \\ &\cong 0.449 - 0.275\frac{t}{\tau_1} + \cdots \cong 0.449\left(1-\frac{t}{1.63\tau_1}+\cdots\right) \cong 0.449\exp\left(-\frac{t}{\tau_{\mathrm{long}}}\right)+\cdots, \end{aligned} \tag{B 9}$$

where

$$\tau_{\text{long}} \equiv 1.63\tau_1 = 615 \text{ years.}$$

An interesting question is: what kind of linear system would possess a Green's function that looks like equation (B 9) for the time-interval $t \ll \tau_1$ but $t \gg \tau_2$?

The answer is the one-tank model, equation (7.2) of the main text, §7

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \kappa\left(E - \frac{C - C_{\text{pre}}}{\tau_{\text{L}}}\right), C_{\text{pre}} = 600. \tag{B 10}$$

This one-tank model with

$$\kappa = 0.449, \quad \tau_{\text{L}} = \kappa\tau_{\text{long}} = 276 \text{ years},$$

will emulate the Sarmiento model in the time range mentioned. In the large τ_{L} limit (when the 'sink' term is omitted), we recover the Constant Airborne Fraction Model of the main text, equation (2.1). The fact that the factor κ is involved in the 'sink' term in equation (7.2) of the main text has puzzled some readers. The derivation above provides the rationale of using τ_{L} instead of τ_{long} as the long-term time-scale.

When t/τ_1 is not small and γ is finite, equation (B 10) can continue to be an adequate emulator for the Sarmiento model if we allow τ_{L} to increase slowly with time to account for the impacts of a slowly rising value of C_{pre}—which is a stand-in for $D(t)$. If we approximate $D(t)$ by $D(t_0)+(t-t_0)(\mathrm{d}D/\mathrm{d}t)_0+\ldots$, we can manipulate equation (B 7) with the help of equation (B 8) as follows:

$$\begin{aligned}
\frac{\mathrm{d}C}{\mathrm{d}t} &\cong k\left\{E - \frac{C - [D(t_0) + (t-t_0)(\mathrm{d}D/\mathrm{d}t)_0 + \cdots]}{\tau_{\text{L}}}\right\} \\
&\cong k\left\{E - \frac{[C - D(t_0)][1 - (t-t_0)\frac{(\mathrm{d}D/\mathrm{d}t)_0}{C - D(t_0)} + \cdots]}{\tau_{\text{L}}}\right\} \\
&\cong k\left\{E - \frac{[C - D(t_0)][1 - (t-t_0)\frac{C(t_0) - D(t_0)}{\gamma\tau_{\text{L}}(C - D(t_0))} + \cdots]}{\tau_{\text{L}}}\right\} \\
&\cong k\left[E - \frac{C - D(t_0)}{\tau_{\text{L}}[1 + (t-t_0)\frac{C(t_0) - D(t_0)}{\gamma\tau_{\text{L}}(C - D(t_0))} + \cdots]}\right] \cong k\left(E - \frac{C - D(t_0)}{\tau^*_{\text{L}}}\right),
\end{aligned} \tag{B 11}$$

where $\tau^*_{\text{L}}(t)$ is the 'derived' time-dependent long-term time-scale, which replaces the constant τ_{L}

$$\tau^*_{\text{L}}(t) \equiv \tau_{\text{L}} + (t-t_0)\frac{C(t_0) - D(t_0)}{\gamma(C - D(t_0))} \cong \tau_{\text{L}} + \frac{t-t_0}{\gamma}. \tag{B 12}$$

The final approximate formula above is an *ad hoc* approximation. The value of τ_{L} can now be interpreted as the value of $\tau^*_{\text{L}}(t)$ at time $t = t_0$. The time dependence of $\tau^*_{\text{L}}(t)$ approximately accounts for the slow rise of $D(t)$ in the next few centuries when γ is finite.

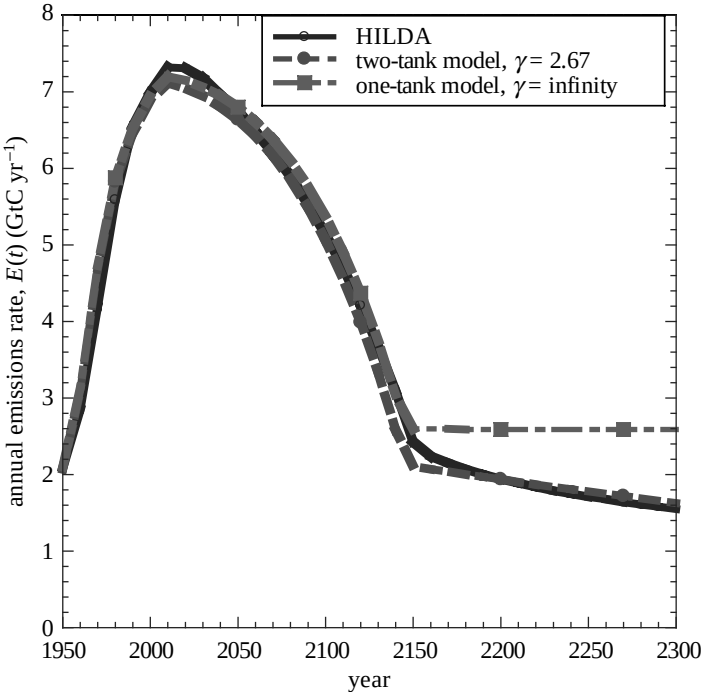

Figure 12. Comparison of emissions trajectories for $C_{\mathrm{stab}}=1155$ GtC (550 p.p.m.). For the two-tank and one-tank models, $\kappa=0.62$ and $\tau_{\mathrm{L}}=225$ years.

The constant values recommended for k and τ_{L} in the main text of this paper, values near 0.5 for κ and near 200 years for τ_{L}, are not precisely the same as the Sarmiento values, but they have the correct order of magnitude. For policy purposes in the next few centuries, they are likely to be good enough. The value of τ_{CPM} is inversely proportional to κ, and the value of E_{stab} is inversely proportional to τ_{L}. The value of γ is not important for policy purposes except when stabilization is near, i.e. when the whole mitigation job is nearly complete and the global warming crisis is nearly over. Its value is then crucial for the estimate of the slow decrease of $E_{\mathrm{stab}}(t)$ after stabilization.

(*f*) *Comparison of two-tank and one-tank models with HILDA*

A well known, science-based carbon cycle model is HILDA (Siegenthaler & Joos 1992; Shaffer & Sarmiento 1995). We use a version of HILDA (without the polar outcrop, i.e. in 1-D mode) to test the credibility of the two-tank and the one-tank models. The specific interesting feature of this version of HILDA is that it includes sophisticated modelling of carbonate buffering of the oceans (Orr *et al.* 2000). It has 68 vertical levels with one-dimensional diffusive exchange between each level.

Three WRE stabilization trajectories for $C(t)$ that are similar to those shown in figure 9 were used (running from year 1750 to 2750, with 450, 550 and 650 p.p.m. as the targets). The emissions trajectories, $E(t)$, were extracted for each case using the HILDA model and the two-tank model. In the latter case, a single set of values of κ, τ_{L} and γ was determined by minimizing the sum of the

mean-square errors for all three cases. The two extracted values of $E(t)$ for the $C_{\mathrm{stab}}=550$ p.p.m. case are shown in figure 12 for the time-interval 1950 to 2300. The HILDA model $E(t)$ is shown as a solid line and the two-tank model $E(t)$ is shown as a dashed line with solid circles. Also shown is the one-tank-model $E(t)$ (obtained by setting $\gamma=$infinity while keeping k and τ_{L} unchanged) as a dashed line with solid squares. It is seen that between 1950 and 2150 the agreement is excellent. The two-tank model is seen to be able to emulate the slow decay of $E_{\mathrm{stab}}(t)$ after 2150, while understandably the one-tank model (with $\gamma=$infinity) does not reproduce this feature. The constant value of E_{stab} of the one-tank model after 2150 is primarily controlled by the value of τ_{L} adopted.

With the same parameters, the two-tank and one-tank models also emulate the $C_{\mathrm{stab}}=450$ and 650 p.p.m. cases adequately. Between 2000 and 2100 when $E(t)$ is steadily dropping, the $E(t)$'s extracted from the two-tank and one-tank models for the 650 p.p.m. case using the same parameters continue to do well, while agreement for the 450 p.p.m. case is less impressive.

(*g*) *Simulations using the one-tank model*

Once an $E(t)$ has been explicitly specified, simulations can be performed (at the spreadsheet level) using the One-tank Model. The One-tank Model slightly undershoots the target C_{stab}, for all mitigation strategies. We have found a way to remove even this small undershoot. We modify equation (3.4), which defines the variable, τ_{CPM}, by replacing the factor of 2 by a variable which we tune. We find empirically that the undershooting, for all interesting cases, can be essentially removed by replacing 2 by 2.2 in equation (3.4).

Appendix C. Notation

A	A_n's are coefficients in Green's functions in appendix B
B	defined by figure 11
C	atmospheric carbon content, in units of GtC
D	defined by figure 11
E	anthropogenic carbon emissions in units of GtC yr^{-1}
H	headroom, defined by equation (3.1)
P	pace, amount of annual reduction of E, in units of GtC yr^{-1} per year
k	dimensionless parameter of Constant Airborne Fraction Model, equation (2.1)
s	dimensionless parameter defined by equation (6.2)
t	time, in units of years
β	cross-section of the B tank relative to the C tank in figure 11
γ	cross-section of the D tank relative to the C tank in figure 11
κ	dimensionless parameter of One-tank Model, equation (7.1)
λ	dimensionless parameter of *ad hoc* Model, equation (2.3)
η	dimensionless time in parabolic mitigation analysis
τ	time constants, in units of years
subscripts	
$2x$	twice the pre-industrial value [$C_{2x}=1200$ GtC]
CPM	constant pace mitigation
PM	parabolic mitigation
0	evaluated at $t=t_0$

pre	evaluated at pre-industrial revolution times [C_{pre}=600 GtC]
stab	value reached when stabilization target is reached
L	long term
S	short term (appendix B only)
long	long term (appendix B only)
superscripts	
max	maximum
min	minimum
*	a variation
acronyms	
CPM	constant pace mitigation, defined in §3
BAU	business as usual, defined in §6*a*

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